C of C Math Meet 2007
MARATHON

Rules:

i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.

ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.

iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.

iv. The judges' decisions will be final.

v. All papers are to be mailed to the following address or submitted electronically to `mathmeet@cofc.edu`

> ***Math Meet (Marathon)***
> ***Department of Mathematics***
> ***College of Charleston***
> ***Robert Scott Small Building / Room 339***
> ***Charleston, SC 29424***

vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .

vii. All entries must be received or postmarked by February 14, 2007.

1. You can easily check that $2007 = 29^2 + 28^2 + 19^2 + 4^2 + 2^2 + 1^2$ is a sum of six squares. Express 2007 as a sum of positive integer squares, using as few squares as possible.

2. Alan throws his six-sided die repeatedly until a 6 appears, and counts the number of throws it takes. Betty throws her six-sided die repeatedly until a 6 appears, and counts the number of throws it takes. What is the probability that Alan and Betty both require the same number of throws?

3. Find conditions on the constants a, b, and c so that the line with equation $ax + by = c$ has two points of intersection with the circle $x^2 + y^2 = 1$.

4. I'm going to choose ten points on a circle and join each pair of points with a chord. This forms a convex ten-sided polygon together with all of its diagonals. What is the maximum number of regions formed inside the decagon by all these chords?

5. Three years ago I bought some stock. The first year, its value increased by p percent. However, the second year, its value decreased by $2p$ percent. But in the third year, its value increased by $3p$ percent. The result is that it now has exactly the same value as when I bought it. What is the value of p?