

**College of Charleston**  
**Math Meet 2007**  
**Written Test – Level 1**

1. In a survey concerning a local issue, 19 percent of doctors voted YES and 95 percent of lawyers voted YES. If all doctors and lawyers voted and 91 percent of all doctors and lawyers voted YES, what is the lawyer to doctor ratio?  
(A) 91 to 19                      (B) 95 to 91                      (C) 18 to 1  
(D) 36 to 1                      (E) none of these
2. There are two cards; one is blue on both sides and the other is blue on one side and green on the other. When one card is chosen and placed on the table, all four possible sides have the same likelihood of appearing face up. If the upper side of the chosen card is blue, then what is the probability that the under-side is also blue?  
(A)  $1/3$                       (B)  $3/4$                       (C)  $1/2$   
(D)  $1/4$                       (E) none of these
3. A circle is inscribed in a square. What percent of the interior of the square is not in the circle?  
(A) between 5 and 10                      (B) between 10 and 15  
(C) between 15 and 20                      (D) between 20 and 25  
(E) none of these
4. If the sum of two distinct numbers is 30 and their product is 10, then the sum of their reciprocals is  
(A) between 0 and 2                      (B) between 2 and 4  
(C) between 4 and 6                      (D) between 6 and 8  
(E) none of these
5. In the expression  $xy^2$ , the values of  $x$  and  $y$  are each decreased by  $1/3$  their value. The value of the new expression will be what fraction of the value of the original expression?  
(A)  $1/3$                       (B)  $4/9$                       (C)  $2/3$   
(D)  $8/27$                       (E) none of these
6. If an equilateral triangle and a regular hexagon have the same perimeter, what is the ratio of the area of the triangle to the area of the hexagon?  
(A) 1 to 6                      (B) 1 to 3                      (C) 2 to 3  
(D) 5 to 6                      (E) none of these
7. The sum of the distances from one vertex of a square with sides of length two to the midpoints of each of the four sides of the square is  
(A)  $4\sqrt{5}$                       (B)  $2 + \sqrt{3}$                       (C)  $2 + 2\sqrt{5}$   
(D)  $1 + \sqrt{5}$                       (E) none of these

8. The average (arithmetic mean) age of a group consisting of nurses and doctors is 40. If the nurses average 35 years and the doctors 50 years, then the ratio of the number of nurses to the number of doctors is
- (A) 2 : 1                      (B) 1 : 2                      (C) 2 : 3  
(D) 3 : 2                      (E) none of these
9. The number  $N_2$  is 25% more than the number  $N_1$ , the number  $N_3$  is 20% more than  $N_2$ , and the number  $N_4$  is  $x\%$  less than  $N_3$ . For what value of  $x$  is  $N_4 = N_1$ ?
- (A) 5                              (B) 45  
(C)  $33\frac{1}{3}$                       (D)  $20\frac{1}{4}$   
(E) None of the above
10. In a certain examination it is noted that the average mark for those passing is 65, while the average of those failing is 35. If the average mark of all participants is 53, what percentage of the participants passed?
- (A) 35                              (B) 40                              (C)  $66\frac{2}{3}$   
(D) 65                              (E) none of these
11. Let  $(a, b)$  denote the set  $\{x \mid a < x < b\}$ . Find the solution set to  $x + 15 < x^2 - x < 8 + x$
- (A) (2, 5)                              (B)  $(2, 4) \cap (3, 5)$   
(C)  $(2, 3) \cup (4, 5)$                       (D)  $(-\infty, 2) \cup (3, \infty)$   
(E) none of these
12. A sphere is inscribed within cube  $A$ , and cube  $B$  is inscribed within the sphere. Which of the following must be equal?
- (A) An edge of  $A$  and the longest diagonal of  $B$   
(B) An edge of  $A$  and the radius of the sphere  
(C) The diameter of the sphere and the longest diagonal of  $A$   
(D) An edge of  $B$  and the diameter of the sphere  
(E) none of these
13. If the price of pens were  $c$  cents a dozen more, you would pay 1 cent more for  $c + 1$  pens than if they were  $c$  cents a dozen less. Given that  $c$  is a positive number, its value is
- (A) between 1 and 3                      (B) between 3 and 5  
(C) between 5 and 7                      (D) between 7 and 9  
(E) none of these

14. The three sides of a triangle have lengths 2, 3, and 4. Find the area of the triangle.

(A)  $\frac{\sqrt{90}}{10}$

(B)  $\frac{3\sqrt{15}}{4}$

(C)  $\frac{9}{2}$

(D)  $3\sqrt{3}$

(E) Not enough information is given to determine the area.

15. Given the following four statements:

I. Exactly one statement is false.

II. Exactly two statements are false.

III. Exactly three statements are false.

IV. Exactly four statements are false

Assume that each statement is either true or false. Among them, the number of false statements is exactly

(A) 0

(B) 1

(C) 2

(D) 3

(E) none of these

16. What is the smallest integral value of  $k$  such that

$$2x(kx - 4) - x^2 + 6 = 0$$

has no real roots?

(A) 5

(B) 4

(C) 3

(D) 2

(E) none of these

17. Triangle ABC has a median AM from vertex A to side BC. Given that the lengths of segments AC, AB, and AM are 8, 4, and 3, respectively, what is the length of side BC?

(A)  $2\sqrt{26}$

(B)  $2\sqrt{31}$

(C) 9

(D)  $4 + 2\sqrt{13}$

(E) none of these

18. A sphere of radius 2 is totally submerged in a cylindrical tank of radius 4. The water level in the tank rises a distance of  $h$ . What is the value of  $h$ ?

(A)  $1/4$

(B)  $3/4$

(C)  $1/3$

(D)  $2/3$

(E) none of these

19. For which real values of  $n$  are the simultaneous equations

$$y = nx + 3$$

$$y = (2n - 1)x + 4$$

satisfied by at least one pair of real numbers  $(x, y)$ ?

- (A) all  $n$  (B) all  $n$  not equal to 0  
(C) all  $n$  not equal to  $1/2$  (D) all  $n$  not equal to 1  
(E) none of these
20. In triangle  $XZY$  with right angle at  $Z$ , altitude  $ZH$  and median  $ZM$  trisect the right angle. If the area of triangle  $ZHM$  is  $x$  square units, then the area of triangle  $XZY$  is how many square units?  
(A)  $3x$  (B)  $4x$  (C)  $6x$   
(D)  $3x\sqrt{3}$  (E) none of these
21. A regular hexagon is inscribed in a circle. If the circle has a radius of 4, what is the area between the hexagon and the circle?  
(A) between 6 and 7 (B) between 7 and 8  
(C) between 8 and 9 (D) between 9 and 10  
(E) none of these
22. For how many values of the coefficient  $a$  do the equations
- $$x^2 + ax + 1 = 0$$
- $$x^2 - x - a = 0$$
- have a common real solution?  
(A) 0 (B) 1 (C) 2  
(D) 3 (E) none of these
23. A polynomial has remainder three when divided by  $x - 1$  and remainder five when divided by  $x - 3$ . The remainder when the polynomial is divided by  $(x - 1)(x - 3)$  is  
(A) 2 (B) 8 (C)  $x - 2$   
(D)  $x + 2$  (E) none of these
24. If  $a$  and  $b$  are prime numbers and  $x^2 - ax + b = 0$  has distinct positive integral solutions, then which of the following statements is true?  
(A)  $a + b$  is prime  
(B) the difference of the solutions is even  
(C)  $b^2 + a$  is not prime  
(D) neither solution is prime  
(E) none of these

25. In a tennis tournament,  $n$  women and  $2n$  men play, and each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number of matches won by men is  $7$  to  $5$ , then  $n$  equals
- (A) 3                                      (B) 8                                      (C) 11  
(D) 16                                      (E) none of these

## 2007 Answers / Level 1 Test

1. C
2. E
3. D
4. B
5. D
6. C
7. C
8. A
9. C
10. E
11. E
12. A
13. A
14. B
15. D
16. D
17. B
18. D
19. D
20. B
21. C
22. B
23. D
24. A
25. A