

College of Charleston
Math Meet 2008
Written Test – Level 2

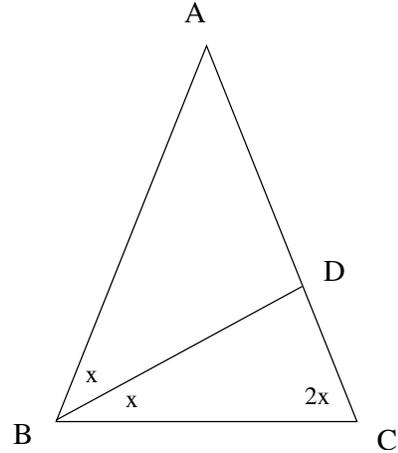
1. Most years have 365 days in them, but every fourth year is a leap year having 366 days. In the leap year 2008, January 1st occurred on a Tuesday. What day of the week will it be on January 1st in the next leap year?
(A) Sunday (B) Monday
(C) Wednesday (D) Thursday
(E) None of the above
2. Let $f(x) = \frac{x^{1/6}x^{1/3}}{(x-20)^{1/4}}$. Find $f(36)$.
(A) $\pm 2\sqrt{3}$ (B) $\pm 2\sqrt{5}$ (C) 18 (D) 6 (E) 3
3. While traveling from his house to his grandmother's house, George fell asleep when he was half of the distance to her house and woke up again before arriving at her house. When he awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire distance had he been asleep?
(A) 1/3 (B) 2/3 (C) 1/4 (D) 2/5 (E) 1/2
4. The U. S. Department of the Interior has classified some species as follows:
 - $T = \{x: x \text{ is a member of threatened species}\}$
 - $E = \{x: x \text{ is a member of an endangered species}\}$
 - $M = \{x: x \text{ is a mammal}\}$Using the symbol \cup to denote the union of sets and the symbol \cap to denote the intersection of sets, express the following in set notation: "The set of all species that are either endangered mammals or threatened mammals."
(A) $(T \cap M) \cup E$ (B) $(T \cup E)$
(C) $(T \cap M) \cap E$ (D) $(T \cup E) \cap M$
(E) none of these
5. What is the probability of getting a sum of sixteen in tossing three standard dice?
(A) 1/6 (B) 1/27
(C) 1/36 (D) 1/12
(E) None of the Above
6. Express the domain of the function $y = \ln(x^2 - 4)$ using interval notation.
(A) $(-\infty, \infty)$
(B) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
(C) $[-2, 2]$
(D) $(-\infty, -2) \cup (2, \infty)$
(E) None of the Above

13. The domain of the function $g(x) = \max\{\sin x, \cos x\}$ is $(-\infty, \infty)$. Find its range.
- (A) $(-1, 1]$ (B) $[0, 1]$ (C) $\left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$
- (D) $\{1\}$ (E) $\left[\frac{-1}{\sqrt{2}}, 1\right]$
14. Find all values of x in $[0, 2\pi]$ which satisfy the equation $\tan(2x) = -\sqrt{3}$.
- (A) $\pi/3, 5\pi/6$
 (B) $4\pi/3, 11\pi/6$
 (C) $5\pi/12, 11\pi/12$
 (D) $\pi/3, 5\pi/6, 4\pi/3, 11\pi/6$
 (E) $5\pi/12, 11\pi/12, 17\pi/12, 23\pi/12$
15. The 2003 inhabitants of an island are divided in two groups: the "truth tellers", who always tell the truth, and the "liars", who always lie. Each person is exactly one of the following: a singer, a soccer player or a fisherman. We ask each inhabitant the following three questions: 1) Are you a singer? 2) Are you a soccer player? 3) Are you a fisherman? 1000 people answer "yes" to the first question, 700 people answer "yes" to the second question, 500 people answer "yes" to the third question. How many "liars" are there on the island?
- (A) 105 (B) 183 (C) 197 (D) 319 (E) 732
16. The function $\frac{x+1}{x^3+1}$ can be written as the sum of an even function and an odd function. Find the even function.
- (A) $\frac{x^4-1}{x^3+1}$ (B) $\frac{x^4+1}{x^6+1}$ (C) $\frac{x^4-1}{x^6+1}$
- (D) $\frac{x^4+1}{x^6-1}$ (E) $\frac{x^4-1}{x^6-1}$
17. What is true about the number $n = 142,857$?
- (A) The number $7n$ is greater than 1,000,000
 (B) $n, 2n, 3n, 4n, 5n$ and $6n$ have the same six digits
 (C) It is the largest prime number
 (D) All of the above
 (E) None of the above

18. Which of these is a solution to the equation

$$2^{2x} - 8 \cdot 2^x = -12?$$

- (A) $1 + \frac{\log 3}{\log 2}$ (B) $\frac{1}{2} \log 6$ (C) $1 + \log \frac{3}{2}$
 (D) $\log 3$ (E) none of these
19. Isosceles triangle ABC has the property that, if D is a point on AC such that BD bisects base angle ABC , then triangles ABC and BCD are similar. If BC has length 1, then AB has length



- (A) $\sqrt{2}$ (B) $\frac{1 + \sqrt{5}}{2}$
 (C) 1 (D) 2
 (E) none of the above
20. If $b > 0$, find the value of $b - a$ so that the domain and range of

$$f(x) = a + b \cos^{-1} x$$

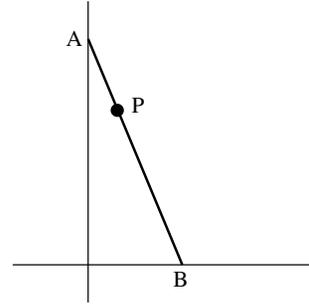
are equal.

- (A) $1 - \frac{1}{\pi}$ (B) $\frac{2}{\pi} + 1$ (C) $1 - \frac{2}{\pi}$ (D) 2
21. How many real numbers x satisfy $|2x - 3| + |x - 3| = |4x - 1|$?
- (A) 1 (B) 2
 (C) 3 (D) 4
 (E) There are no solutions.

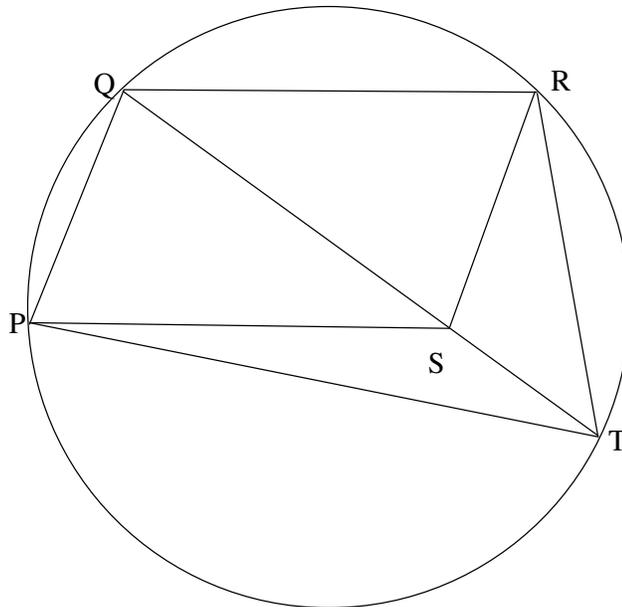
22. A prisoner is given two bowls, ten red tokens, and ten blue tokens. He is instructed to distribute the tokens into the two bowls in any fashion that he chooses provided that neither bowl is empty and all twenty tokens are distributed into the bowls. The next day, the warden will randomly choose a bowl and then randomly choose a token from that bowl. If the chosen token is blue, then the prisoner will be set free. If the prisoner distributes the tokens in an optimal manner, his probability of freedom is which of the following?

- (A) $13/21$ (B) $8/11$ (C) $9/13$ (D) $14/19$ (E) $12/29$

23. As points A and B slide along the axes, point P on the line segment between them remains a fixed distance from A and a fixed distance from B. The path traced out by P is part of what kind of figure?



- (A) straight line (B) circle (C) ellipse
 (D) hyperbola (E) parabola
24. Vertices P, Q, and R of parallelogram PQRS lie on a circle, while vertex S lies inside the circle (see diagram). Line QS intersects the circle at point T. If angle PTR is 70 degrees and angle SPT is 15 degrees, what is angle TRS in degrees?



- (A) 15 (B) 20 (C) 25 (D) 30 (E) 40
25. There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on the surface of each ball which is exactly 10 inches from each wall which that ball touches and 10 inches from the floor, then what is the sum of the radii of the balls?
- (A) 15 inches (B) 20 inches
 (C) 25 inches (D) 30 inches
 (E) not enough information

2008 Answers / Level 2 Test

1. A
2. E
3. A
4. D
5. C
6. D
7. D
8. B
9. E
10. A
11. A
12. D
13. E
14. D
15. C
16. E
17. B
18. A
19. B
20. B
21. B
22. D
23. C
24. C
25. D

