

# C of C Math Meet 2009 MARATHON

## **Rules:**

- i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- iv. The judges' decisions will be final.
- v. All papers are to be mailed to the following address or submitted electronically to [mathmeet@cofc.edu](mailto:mathmeet@cofc.edu)

**Math Meet (Marathon)**  
**Department of Mathematics**  
**College of Charleston**  
**Robert Scott Small Building / Room 339**  
**Charleston, SC 29424**

- vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .
- vii. All entries must be received or postmarked by February 14, 2009.

## **The Questions:**

1. How many integers  $x$  between 0 and 2009 have the property that the line segment joining  $(0, 0)$  and  $(x, 2009)$  does not contain any other points  $(a, b)$  whose coordinates are both integers?
2. Find the longest sequence of consecutive positive integers whose sum is 2009.
3. The number 2009 has base 10 digits which sum to 11. For what other bases  $b$  do the sum of digits in the base  $b$  representation of 2009 add up to 11?
4. A right triangle has integer legs and hypotenuse 2009. What is its perimeter?
5. Two circles in the first quadrant are tangent to each other and both tangent to the  $x$ -axis and to the line  $y = mx$ , where  $m$  is a positive constant. Find the ratio of the radius of the larger circle to the radius of the smaller circle, as a function of  $m$ .