

## C of C Math Marathon 2010

### **Rules:**

- i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- iv. The judges' decisions will be final.
- v. All papers are to be mailed to the following address or submitted electronically to [mathmeet@cofc.edu](mailto:mathmeet@cofc.edu):  
Math Meet (Marathon)  
Department of Mathematics  
College of Charleston  
Robert Scott Small Building / Room 339  
Charleston, SC 29424
- vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .
- vii. All entries must be received or postmarked by February 12, 2010.

### **The Questions:**

1. A right triangle has sides of integer length and perimeter 2010 units. The length of the shortest side is equal to one-fifth of the sum of the lengths of the other two sides. What is the area of the triangle?
2. A large circle is just large enough to contain in its interior three non-overlapping, mutually tangent smaller circles, each of which has area equal to one square centimeter. It is also just large enough to contain in its interior four non-overlapping smaller circles, each of the same area. What is the combined area of these four smaller circles?
3. A rectangular box has sides of integer length and surface area equal to 2010 square units. What is the maximum volume it could have?
4. Player A chooses a number  $n$  at random from the set  $\{1, 2, \dots, 2010\}$ . A game is then played with a deck of cards labeled with the integers of absolute value less than or equal to  $n$ , one card for each such integer. Player B selects a card at random from this deck and keeps it. Player A attempts to guess player B's card. If Player A correctly guesses Player B's card, then Player A selects a card from the remaining deck and keeps it. Player B then selects another card at random from the remaining deck, and Player A attempts to guess Player B's second card. What is the probability that Player A correctly guesses both of Player B's cards?
5. This year's Math Meet will be held on 2/20/2010. Find the number of distinct arrangements of the digits 2202010. (Note that, for example, interchanging the first two digits does not produce a distinct arrangement).