

College of Charleston Math Meet 2010 Written Test – Level 2

- The hypotenuse of a right triangle measures 13 inches. The sum of the lengths of the legs is 17 inches. What is the area of the triangle?
(A) 40 square inches (B) 26 square inches (C) 35 square inches
(D) 30 square inches (E) 42 square inches
- An ATM (an automated banking machine) is able to distribute five dollar bills, ten dollar bills and twenty dollar bills. By mistake, an employee puts twenty dollar bills in the five dollar bill slot. As a result, some people get more money out of the machine than they *should* when they make a withdrawal. Which of these amounts could possibly be the amount of money that the bank lost due to this mistake?
(A) \$9223 (B) \$101 (C) \$99 (D) \$315 (E) \$522
- An equilateral triangle ACD with each side length x units, is inscribed in a circle. What is the length of the radius of this circle?

- (A) $\frac{x\sqrt{3}}{3}$ (B) $4x\sqrt{3}$ (C) $7x\sqrt{3}$
(D) $\frac{x\sqrt{3}}{2}$ (E) none of these

- Which of these is equal to

$$\left(\frac{x^2 + 1}{x}\right) \left(\frac{y^2 + 1}{y}\right) + \left(\frac{x^2 - 1}{y}\right) \left(\frac{y^2 - 1}{x}\right)$$

when $xy \neq 0$?

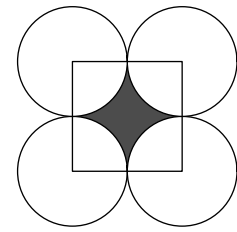
- (A) 1 (B) $2xy$ (C) $2x^2y^2 + 2$
(D) $2xy + \frac{2}{xy}$ (E) $\frac{2x}{y} + \frac{2y}{x}$
- A carpenter agrees to work under the condition that he is to be paid \$5.50 every day he works and he must pay \$6.60 every day he does not work. At the end of 30 days he has earned \$7.70. How many days did he work?
(A) 14 (B) 15 (C) 16
(D) 17 (E) None of the above
 - Let x be the solution to the equation

$$\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3.$$

If x is written as a fraction in reduced form, then the numerator is:

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. In the figure, each circle has radius of 4. Each corner of the square is the center of a circle. The circles are tangent to each other as shown. The area of the shaded region is closest to



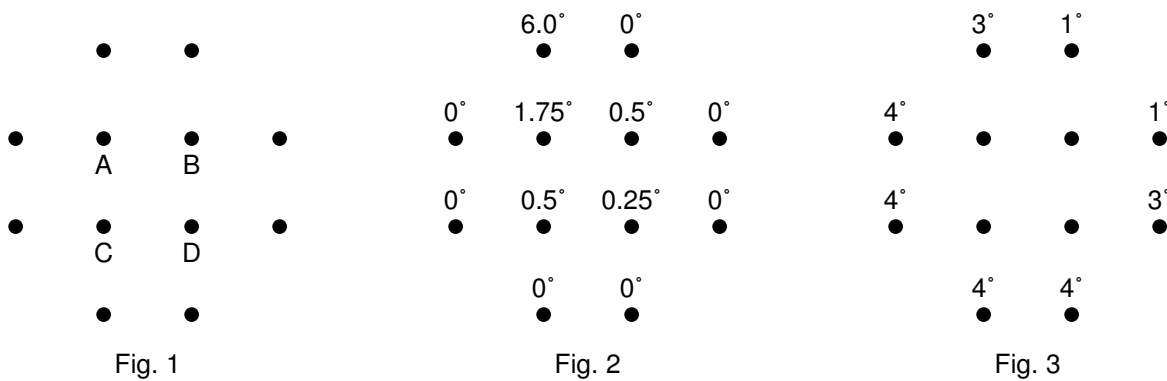
- (A) 3.4 (B) 13.7 (C) 48.0 (D) 50.2 (E) 114.2

8. In a group of cows and chickens, the number of legs is eighty-four more than twice the number of heads. How many cows are there?

- (A) 84 (B) 42 (C) 48
(D) 36 (E) None of the above

9. When we measure the temperature at the 12 points in Fig. 1, we find that the temperature at each interior point A, B, C, or D is the average of the temperatures at its four nearest neighbors. See, for example, the temperatures in Fig. 2. If the temperatures in Fig. 3 follow the same rule, what is the average of the temperatures at the 12 points in Fig. 3?

The correct answer (rounded to two decimal places) is:



- (A) 2.00 (B) 2.25 (C) 2.75 (D) 3.00 (E) 3.33

10. A club of 15 members must choose a delegation of 4 members to serve at a convention. Ewa and Bob each refuse to serve in the delegation unless the other one also serves. How many delegations are possible?

- (A) 793 (B) 806 (C) 819
(D) 832 (E) None of the above

11. The coordinates of the point on the line $y = 2x + 1$ that is closest to the point $(4, 2)$ are:

- (A) $(8/5, 21/5)$ (B) $(6/5, 17/5)$ (C) $(-2/3, -1/3)$
(D) $(4/3, 11/3)$ (E) none of the above

12. Find the graph of the inequality $\frac{y}{x} < \frac{y+1}{x+1}$. (Assume, as usual, that the x -axis is horizontal and the y -axis is vertical.)

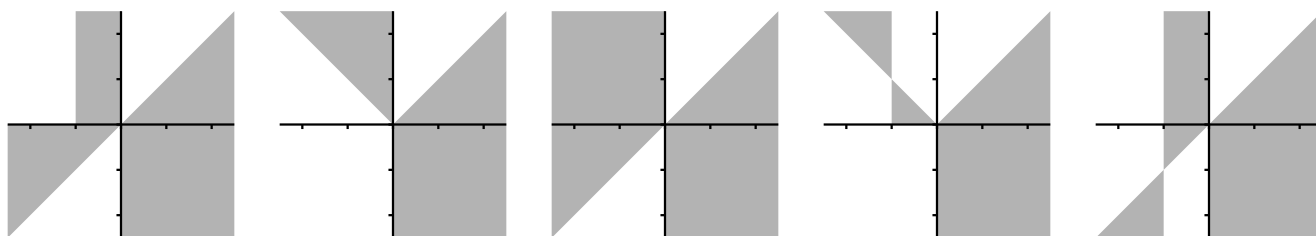


figure a

figure b

figure c

figure d

figure e

- (A) figure a (B) figure b (C) figure c (D) figure d (E) figure e

13. A magician had a magic bag that doubled the amount of money put into it. He agreed to let a man borrow the bag if the man gave the magician \$16 immediately after each doubling. Each time the man used the bag he put in all the money he had; but after the fourth time he used the bag and paid the magician, his money was completely gone. With how much money did he begin?
- (A) \$16 (B) \$64 (C) \$30 (D) \$15 (E) \$13
14. A parade is two miles long and moves with constant speed. A person who is at the end starts walking towards the front of the parade, moves forward with a constant speed until he reaches the front, and then turns around and walks at the same speed until he reaches the end of the parade. By this time the parade has moved two miles forward. What distance has the person walked?
- (A) 4 (B) 2 (C) $2(1 + \sqrt{3})$
 (D) $2(1 + \sqrt{2})$ (E) $3(1 + \sqrt{2})$
15. Let k be a positive number not equal to 1. If

$$(\log_k x)(\log_5 k) = \frac{5}{2},$$

find x .

- (A) 5 (B) 25 (C) $25\sqrt{5}$ (D) $25\sqrt{2}$ (E) $5\sqrt{2}$

16. Among the 6 planes containing the 6 faces of a cube, how many pairs of mutually perpendicular planes can you find?
- (A) 3 (B) 6 (C) 8 (D) 12 (E) 24
17. A man and a woman have the same birthday. When he was as old as she is now, the man was twice as old as the woman. When she becomes as old as he is now, the sum of their ages will be 119. How old is the man now?
- (A) 55 (B) 61 (C) 51
 (D) 45 (E) None of the above.

18. For any two real numbers a and b , define their “diamond product” to be

$$a \diamond b = 4a^2 + 4b^2 - a^2b^2.$$

Which of the following statements are true?

- I. The diamond product satisfies the law of commutativity.
- II. The diamond product satisfies the distributive law (with respect to ordinary addition of numbers).
- III. The diamond product satisfies the associative law.
- IV. The equation $c \diamond x = 4c^2$ has one solution for most values of c , but infinitely many for others.
- V. There are always exactly two solutions to the equation $c \diamond x = c$ no matter what number c is.

- (A) Only I, II and III are true. (B) Only II and V are true.
(C) Only I and IV are true. (D) All of the statements are true.
(E) None of the statements are true.

19. Which of these is a valid description of the function

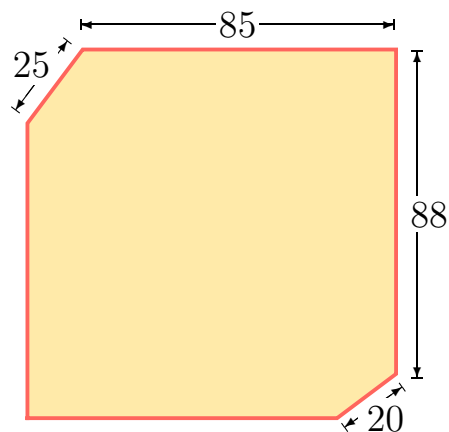
$$f(x) = \frac{\sin(2x)}{2 \sin(x)}?$$

- (A) It has the same value as the function $\cos(x)$ at any x in the domain of f .
- (B) It is a constant function.
- (C) It takes only positive values.
- (D) This is just another way to write the function $\sin^2(x)$.
- (E) It gives the ratio of two sides of any triangle with one angle equal to x .

20. Six coins are drawn, without replacement, from a bag that contains 2 pennies, 4 nickels, and 6 dimes. Each coin has an equally likely chance of being chosen and there are 924 ways that six coins can be drawn. What is the probability that the value of the six drawn coins is at least fifty cents?

- (A) $\frac{22}{924}$ (B) $\frac{36}{924}$ (C) $\frac{127}{924}$
(D) $\frac{540}{924}$ (E) none of these

21. Two corners were cut off a 100×100 square resulting in the hexagon shown in the figure. Four of the sides have the lengths indicated by the labels. What is the total perimeter of this shape?



- (A) 368 (B) 372 (C) 378 (D) 382 (E) 388
22. Which whole number is nearest in value to the sum $\frac{2008}{2999} + \frac{8001}{5998} + \frac{2001}{3999}$?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
23. Express q as a function of p , given that one root of $x^2 + px + q = 0$ is twice the other.
- (A) $q = \frac{p^2}{2}$ (B) $q = \left(\frac{2}{9}\right)p^2$ (C) $q = \frac{p^2}{4}$
- (D) $q = \left(\frac{2}{3}\right)p^2$ (E) None of the Above
24. Let S be the set of all numbers which are the sum of the squares of three consecutive integers. Then we can say that
- (A) No member of S is divisible by 2
 (B) No member of S is divisible by 3
 (C) No member of S is divisible by 5
 (D) No member of S is divisible by 7
 (E) For any prime number N there is an element of S divisible by N
25. A virus has infected Jack's computer, and is erasing increasing amounts of its memory. Every minute, the virus increases the size of the erased region by a factor of four, and also erases one more byte. Jack shuts down his computer at a moment when, if one tripled the erased amount and erased one more byte, the entire $2.5Gb = 2.5 \times 2^{30}$ bytes of memory would be erased. How large was the erased region 14 minutes earlier than the moment Jack shuts down his computer?
- (A) 1 byte (B) 2 bytes (C) 3 bytes (D) 4 bytes (E) 16 bytes

2010 Answers / Level 2 Test

1. D
2. D
3. A
4. D
5. D
6. E
7. B
8. B
9. D
10. A
11. B
12. E
13. D
14. D
15. C
16. D
17. C
18. C
19. A
20. C
21. D
22. C
23. B
24. B
25. C