C of C Math Marathon 2011

Rules:

i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.

ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.

iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.

iv. The judges' decisions will be final.

v. All papers are to be mailed to the following address or submitted electronically to `mathmeet@cofc.edu`:

Math Meet (Marathon)
Department of Mathematics
College of Charleston
Robert Scott Small Building / Room 339
Charleston, SC 29424

vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .

vii. All entries must be received or postmarked by February 14, 2011.

The Questions:

1. I have a collection of pennies which I have divided into two square arrays. One of the square arrays contains 2011 pennies more than the other. How many pennies are in my collection?
2. Each turn in a game begins with Alice throwing a six-sided die; if she throws a 6, she wins. If not, Bob tosses a coin; he wins if it lands on heads. This process repeats until one of them wins. What is the probability that Alice wins?
3. There are three circles in the first quadrant, whose radii are 1, r, and R units, respectively, with r and R larger than 1. The three circles are each tangent to the x-axis at distinct points, and each circle is tangent to each of the other two circles at distinct points. Find the radius R as a function of the radius r.
4. A geometric sequence begins $\sqrt[7]{12}$, $\sqrt[3]{18}$,.... Find the first term of this sequence which is an integer.
5. The function
$$f(x) = \frac{x+1}{x+2}$$
is defined on the interval $(-2, \infty)$; however, the composite function $f \circ f$ is not defined on this interval. Find the smallest value of t so that all of the functions f, $f \circ f$, $f \circ f \circ f$, and in general
$$\underbrace{f \circ f \circ \cdots \circ f}_{n \text{ copies}},$$
for every positive integer n, are defined on the interval (t, ∞).