College of Charleston
Math Meet 2011
Written Test – Level 2

1. A spinner has an equal probability of landing on each of seven spaces, labeled with the numbers one through seven. You are going to be playing a game in which you will spin it 70 times, each time betting in advance on whether it will land on an even or an odd number. Which strategy should you adopt in order to win the most times?

   (A) There is no optimal strategy because it is random.

   (B) Bet *odd* exactly 40 times and *even* exactly 30 times.

   (C) Bet *odd* 35 times and *even* 35 times.

   (D) Bet *even* only if the previous spin landed on an *odd* number.

   (E) Bet on *odd* every time.

2. Which of these complex numbers is equal to $\dfrac{1}{3+2i}$?

   (A) $\dfrac{1}{3}+\dfrac{1}{2}i$ (B) $\dfrac{1}{2}-\dfrac{1}{3}i$ (C) $\dfrac{3}{5}+\dfrac{2}{5}i$

   (D) $\dfrac{3}{10}+\dfrac{1}{5}i$ (E) $\dfrac{3}{13}-\dfrac{2}{13}i$

3. Let $f(x)$ be the function defined by the formula $f(x)=\dfrac{|x|}{x}$. Which of these statements about the domain and range of f is true?

   (A) The range of f is a set containing only two numbers.

   (B) The domain of f is all real numbers.

   (C) The domain is a subset of the range.

   (D) The range contains only positive numbers.

   (E) The domain contains only positive numbers.

4. The number 2011 can be written in binary form as 11111011011 using exactly 2 zeroes and 9 ones. Find the sum of the three smallest integers that are larger than 2011 and can also be written in binary form with 2 zeroes and 9 ones.

   (A) 6042 (B) 6050 (C) 6057 (D) 6065 (E) 6082

5. What is the units digit of $3^{2011}\times 7^{2010}$?

   (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

6. What is the probability that the eighth toss of a coin will produce the second tail?

   (A) $\dfrac{1}{4}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{7}{256}$ (D) $\dfrac{3}{128}$ (E) $\dfrac{7}{128}$

7. In the interval $[0,2\pi]$, how many solutions are there to the equation $\cos 2x+\cos 3x=0$?

   (A) none (B) 1 (C) 2 (D) 3 (E) 5

8. Which of these angles (measured in radians) has a tangent greater than zero and a cosine less than zero?

   (A) π (B) $-\pi/82$ (C) $39\pi/4$ (D) $27\pi/4$ (E) $29\pi/4$

9. A piece of wire 12 inches long is going to be cut into two pieces. One of the pieces will be bent into a perfect square and the other into a perfect circle. If the piece used to make the circle is x inches long, what is the total area contained in the two geometric figures?

(A) $\dfrac{\pi x^2 + 4x^2 - 24\pi x + 144\pi}{16\pi}$

(B) $\dfrac{4\pi x^2 + x^2 - 60\pi x + 36x + 144\pi}{16\pi}$

(C) $\dfrac{\pi x^2 - x^2 - 24\pi x + 60x + 144\pi}{16\pi}$

(D) $\dfrac{\pi x^2 - 24\pi x + 48x + 144\pi}{16\pi}$

(E) $\dfrac{2x^2 - 12\pi x + 24x + 144\pi}{16\pi}$

10. What is the domain of the function $\log_{10}(x^4 + 2x^2)$?

(A) $(-\infty, -10) \cup (10, \infty)$ (all numbers with absolute value bigger than 10)

(B) $(-\infty, 0) \cup (0, \infty)$ (all non-zero numbers)

(C) $(0, \infty)$ (all positive numbers)

(D) $[0, \infty)$ (all non-negative numbers)

(E) $(-\infty, -\ln(10)) \cup (-\ln(10), 0) \cup (0, \ln(10)) \cup (\ln(10), \infty)$

11. Which of these choices lists the numbers in *increasing* order?

(A) $2\sqrt{20}, 3\sqrt{5}, 9, 3\pi$ (B) $9, 3\pi, 3\sqrt{5}, 2\sqrt{20}$

(C) $3\sqrt{5}, 9, 3\pi, 2\sqrt{20}$ (D) $3\sqrt{5}, 2\sqrt{20}, 9, 3\pi$

(E) $2\sqrt{20}, 9, 3\sqrt{5}, 3\pi$

12. Which of the following is equal to

$$\ln(1 + x^1) + \ln(1 + x^2) + \ln(1 + x^4) + \ln(1 + x^8) + \cdots + \ln(1 + x^{32})?$$

(A) $\ln(1 + x + x^2 + x^3 + \cdots + x^{63})$

(B) $\ln(1 + x^{64})$

(C) $\ln(1 + x + x^2 + x^4 + x^8 + \cdots + x^{32})$

(D) $32\ln(1 + x)$

(E) none of these

13. In what follows, a 'perfect square' refers to the square of a whole number. Which of the following statements is true?

I. *Every* perfect square can be written as a sum of two odd numbers.
II. If a perfect square is an even number then you still have a perfect square after you divide it by 4.
III. If you compute the sum of all of the odd numbers between 1 and any given positive number you *always* get a perfect square.

(A) Only I is true. (B) Only II is true.

(C) Only III is true. (D) Exactly two of them are true.

(E) All three are false.

14. What real number x satisfies the equation $3^{2x+1} = 2(5^x)$?

(A) $\dfrac{\ln(2) - \ln(3)}{2\ln(3) - \ln(5)}$ (B) $\dfrac{\ln(2) + \ln(3)}{2\ln(3) - \ln(5)}$ (C) $\dfrac{\ln(2) - \ln(3)}{2\ln(3) + \ln(5)}$

(D) $\dfrac{\ln(2) - 2\ln(3)}{\ln(3) + \ln(5)}$ (E) $\dfrac{\ln(2) + 2\ln(3)}{2\ln(3) - \ln(5)}$

15. John and Joe from Nebraska consult the Mathematical Oracle of Omaha to learn whether they have pairs (x, y) of lucky (real) numbers. In order to determine the pairs of lucky numbers, the Oracle asks John and Joe to each write down his day of birth (d) and month of birth (m), and then solves the system

$$\begin{cases} 13x - y = 181 \\ dx - my = 362. \end{cases}$$

The Oracle's response is that Joe has no pair of lucky numbers, while John has an infinite number of pairs of lucky numbers. Which of the following statements is true?

(A) John and Joe were both born in the spring.

(B) John and Joe were both born in the summer.

(C) John and Joe were both born in the fall.

(D) John and Joe were both born in the winter.

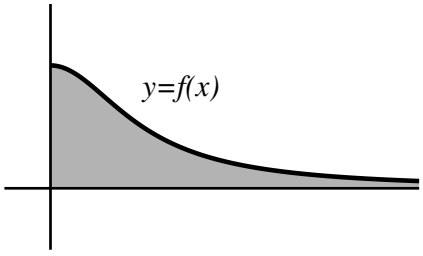
(E) John and Joe were born in different seasons.

16. A pyramid has its peak directly over the center of its base, which is a square with side length 10 cm. If its height is 5 cm, what is the surface area of the pyramid in cm^2?

(A) $25\sqrt{2}$ (B) $50\sqrt{2}$ (C) $100\sqrt{2}$

(D) $100 + 100\sqrt{2}$ (E) $100 + 200\sqrt{2}$

17. $f(x)$ is the nonnegative function defined on $[0, \infty)$ whose graph is shown in the figure. The shaded area trapped between the graph of $f(x)$ and the x-axis is 4. What's the area trapped between the x-axis and the graph of $2f(2x)$?



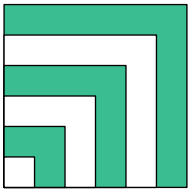
(A) 16 (B) 8 (C) 4 (D) 2 (E) 1

18. If the cosine of an acute angle is $1/3$, what is the cosine of an angle half that size?

(A) $\frac{1}{6}$ (B) $\frac{2}{3}$ (C) $\sqrt{\frac{1}{6}}$

(D) $\sqrt{\frac{2}{3}}$ (E) $\sqrt{\frac{1}{6}+\frac{2}{3}}$

19. What fraction of this square is shaded?



(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

20. When a sphere of radius 1 is totally submerged in a cylindrical tank of radius 2, the water level in the tank rises a distance of h. What is the value of h?

(A) $1/12$ (B) $1/4$ (C) $1/3$

(D) $2/3$ (E) none of these

21. Find $p(-1)$ if p is the cubic polynomial which takes the following values:

x	0	1	2	3
$p(x)$	2	1	0	1

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2

22. Simplify $\dfrac{\sin\theta\sec\theta+\cos\theta\csc\theta}{(\sec\theta-1)(\sec\theta+1)}$.

(A) 1 (B) $\cos\theta\sin\theta$ (C) $\dfrac{\cos\theta}{\sin^3\theta}$

(D) $\dfrac{\sin\theta}{\cos^3\theta}$ (E) $\cos^2\theta\sin\theta$

23. How many times do the graphs of the functions

$$f(x)=\cos(x)-2 \quad \text{and} \quad g(x)=x(x+1)-(x-1)$$

intersect each other?

(A) zero (B) once

(C) twice (D) nine times

(E) infinitely many times

24. How many positive integers between 100 and 999 (inclusive) do not contain three identical digits?

(A) 889 (B) 890 (C) 891 (D) 892 (E) 991

25. Let us use the symbol "$\oplus$" between two numbers to represent the maximum of the two numbers. So, for instance, $2 \oplus 1 = 2$ and $(-50) \oplus 19 = 19$. Which of these statements is true for *all* real numbers a, b and c?

I. $a \oplus b = b \oplus a$
II. $(a \oplus b) \oplus c = a \oplus (b \oplus c)$
III. $c \times (a \oplus b) = (c \times a) \oplus (c \times b)$

(A) Only I is true.

(B) Only II is true.

(C) Only I and II are true.

(D) All three are true.

(E) None of these statements is true for all values of a, b and c.

2011 Answers / Level 2 Test

1. E
2. E
3. A
4. B
5. B
6. C
7. E
8. E
9. A
10. B
11. D
12. A
13. D
14. A
15. D
16. D
17. C
18. D
19. C
20. C
21. B
22. C
23. A
24. C
25. C