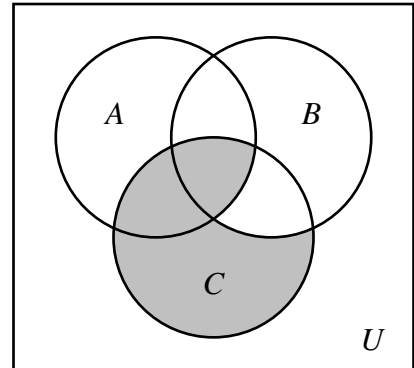


College of Charleston Math Meet 2011 Written Test – Level 3

1. If A , B , and C are subsets of the set U , then
- $A \cap B$ is the set of all elements that belong to both A and B ,
 - $A \cup B$ is the set of all elements that belong to A or B (or both), and
 - A^c is the set of all elements of U that are **not** in A .

In the diagram, A , B and C are represented by circles and U is represented by a square.



Which of the following expressions describes the region shaded in the diagram?

- (A) $(A \cup B^c) \cap C$ (B) $(A \cup B) \cup C^c$ (C) $(A^c \cap B^c) \cup C$
 (D) $(A \cap B) \cap C$ (E) $(A^c \cup B) \cup C^c$

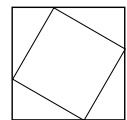
2. Let ρ be a rotation about the origin by 51° clockwise. What is the minimal number of times one must repeat ρ before every point returns to where it started?

- (A) 30 (B) 60 (C) 120
 (D) 360 (E) none of these

3. Find the complex number z if $(5 - 7i)z = 13 + 4i$.

- (A) $\frac{1}{2} + \frac{3}{2}i$ (B) $\frac{3}{2} + \frac{1}{2}i$ (C) $\frac{1}{2} - \frac{3}{2}i$ (D) $\frac{3}{2} - \frac{1}{2}i$ (E) $\frac{1}{2} - \frac{1}{2}i$

4. Let S_1 be a square with side length 1 meter. Rotate a copy of S_1 by 60° and then shrink it so its vertices lie on S_1 . (See figure at right.) Call this new square S_2 . What is the area of S_2 in square meters?



- (A) $\sqrt{3} - 1$ (B) $4 - 2\sqrt{3}$ (C) $\frac{3}{4}$
 (D) $\frac{2}{\sqrt{3} - 1}$ (E) none of these

5. Let $f(x) = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$. Compute $f^{(4)}(3)$.

- (A) $-\frac{51}{32}$ (B) 0 (C) $\frac{1}{2}$
 (D) $\frac{45}{8}$ (E) none of these

6. How many real solutions are there to $\tan^{-1}(\tan x) = \frac{\pi}{2} - x$?

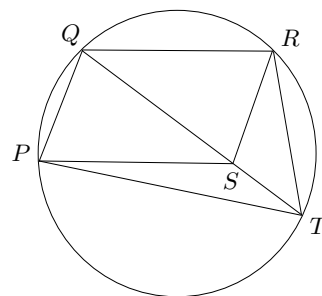
- (A) there are no real solutions (B) 1
 (C) 2 (D) 5
 (E) infinitely many

7. Which of the following is equal to

$$\ln(1 + x^1) + \ln(1 + x^2) + \ln(1 + x^4) + \ln(1 + x^8) + \cdots + \ln(1 + x^{32})?$$

- (A) $\ln(1 + x + x^2 + x^3 + \cdots + x^{63})$
- (B) $\ln(1 + x^{64})$
- (C) $\ln(1 + x + x^2 + x^4 + x^8 + \cdots + x^{32})$
- (D) $32 \ln(1 + x)$
- (E) none of these

8. Vertices P , Q , and R of parallelogram $PQRS$ lie on a circle, while vertex S lies inside the circle (see diagram). Line QS intersects the circle at point T . If angle PTR is 60 degrees and angle SPT is 25 degrees, what is angle TRS in degrees?



- (A) 20
- (B) 25
- (C) 30
- (D) 35
- (E) 40

9. $(1 + x)^9$ can be expanded as $\sum_{k=0}^9 a_k x^k$. How many of the a_k are divisible by 9?

- (A) none of them are divisible by 9
- (B) 3
- (C) 5
- (D) 6
- (E) 8

10. A bug starts at the origin of the xy -plane. Every second it goes up or right one unit, each with probability 0.5. At one instant, you notice that the bug is at the point (3,4). What is the probability that the bug went through the point (3,3)?

- (A) $\frac{3}{5}$
- (B) $\frac{1}{2}$
- (C) $\frac{3}{7}$
- (D) $\frac{2}{5}$
- (E) $\frac{4}{7}$

11. Farmer Mary keeps four kinds of animals: cows, chickens, ducks, and pigs. Next year, she would like to have 14 cows and chickens (that is, her cows and chickens should total 14), 12 chickens and ducks, 8 ducks and pigs, and 10 pigs and cows. What's the total number of animals must she have altogether?

- (A) 22
- (B) 32
- (C) 42
- (D) It's impossible for her to do this.
- (E) It's possible for her to do this, but there's more than one possible total.

12. Find a polynomial which is a factor of $(x^2 + 1)^{1005} + x^{2010}$.

- (A) $x^2 - 1$
- (B) $x^2 + \frac{1}{4}$
- (C) $2x^2 + 1$
- (D) $x^2 + 4$
- (E) $x^2 + 2$

13. "Flops" is a single-player game played with a deck of cards. The 2010 Flops World Cup was 100-day nationwide tournament in which 39,840 individual games were played. (Here, "game" refers to one player earning one score.) The highest score was 400, and the lowest score was 2. If all flops scores are integers, which of the following statements must be true?
- Every possible score between 2 and 400 was attained at least once in the tournament.
 - On at least one day during the tournament, two games were played with the same score.
 - On at least one day during the tournament, two games were played by players born on the same day of the year (possibly the same player).
- (A) I only.
 (B) II only.
 (C) III only.
 (D) II and III only.
 (E) None of the three statements has to be true.
14. Suppose $e(x)$ is an even function, $o(x)$ is an odd function, and the function $n(x)$ is neither even nor odd. How many of the following functions must be even?
- $n(e(x))$
 - $e(n(x))$
 - $e(o(x))$
 - $n(o(x)) + n(-o(x))$
 - $o(x) \cdot (n(x) - n(-x))$
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
15. It takes 10 minutes to fill my tub with the hot and cold water both on and the drain unplugged. It takes 30 minutes to fill my tub with the cold water on and the drain unplugged. It takes 60 minutes to fill my tub with the hot water on and the drain unplugged. How many minutes does it take to fill my tub with both the hot and cold water on and the drain plugged? The correct answer is nearest
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
16. Which of these numbers is the *largest*?
- (A) $15\sqrt{3}$ (B) $6\sqrt{26}$ (C) $\sqrt{882}$ (D) $5\sqrt{40}$ (E) $4\sqrt{50}$
17. What is $9395 \times 9395 - 9393 \times 9397$?
- (A) 17^2 (B) -2 (C) 49 (D) -12 (E) 4
18. When the number $.481818181\dots$ (with the 81's repeating forever) is written as a fraction in lowest terms, by how much does the denominator exceed the numerator?
- (A) 13 (B) 14 (C) 29 (D) 57 (E) 126
19. The light rays from a spotlight spread out to form a solid right circular cone, with the light rays making an angle of up to 30 degrees with the centerline of the beam. If the spotlight is 10 feet above the floor, and the centerline of the beam hits the floor at a 60-degree angle of inclination, what is the area (in square feet) that the spotlight covers on the floor?
- (A) $100\pi\sqrt{3}$ (B) $25\pi\sqrt{6}$ (C) $\frac{100\pi}{\sqrt{3}}$
 (D) $25\pi\sqrt{7}$ (E) $\frac{400\pi}{9}$

20. $\int_1^2 \sqrt{4-x^2} dx =$

- (A) $\frac{4\pi}{3} - \frac{1}{2}$ (B) $\pi - \frac{\sqrt{2}}{2}$ (C) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$
 (D) $\frac{\pi}{3} - 1$ (E) $\frac{10\pi}{3} - \frac{\sqrt{5}}{2}$

21. The graph of the polar equation $r = -2 \sin \theta$ is

- (A) a straight line. (B) a parabola. (C) a circle.
 (D) a four-leafed rose. (E) a cardioid.

22. Claire is a little dizzy from too much sun at the beach and she starts walking in a strange way:

- she takes one step forward,
- she turns 90° to her right and then takes two steps forward,
- she turns 90° to her right and then takes one step forward,
- she turns 90° to her left and then takes one step backward,
- she starts all over again.

Each step is 1 yard. After 186 steps, Claire passes out. How many yards from where she started does Claire end up?

- (A) 186 (B) 1 (C) 2 (D) $\sqrt{2}$ (E) $\sqrt{5}$

23. Find $A + B + C$ if A , B , and C are constants and

$$\frac{1}{(x-1)(x+1)(x-2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2}$$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

24. When the polynomial $p(x)$ is divided by $x^2 - 1$, the remainder is $x + 2$. When $p(x)$ is divided by $x^2 - 4$, the remainder is $x + 1$. Find the remainder when $p(x)$ is divided by $(x-1)(x-2)$.

- (A) $(x+1)(x+2)$ (B) $x+1$ (C) $x-1$
 (D) $x+2$ (E) 3

25. Evaluate $\lim_{x \rightarrow 0^+} e^{i \cos^{-1}((\ln x)/(1-\ln x))}$, where $i = \sqrt{-1}$.

- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

2011 Answers / Level 3 Test

1. A
2. C
3. A
4. B
5. D
6. C
7. A
8. D
9. D
10. E
11. A
12. C
13. C
14. D
15. D
16. D
17. E
18. D
19. B
20. C
21. C
22. C
23. C
24. E
25. B