

C of C Math Marathon 2012

Rules:

- i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- iv. The judges' decisions will be final.
- v. All papers are to be mailed to the following address or submitted electronically to mathmeet@cofc.edu:
Math Meet (Marathon)
Department of Mathematics
College of Charleston
Robert Scott Small Building / Room 339
Charleston, SC 29424
- vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .
- vii. All entries must be received or postmarked by February 11, 2012.

The Questions:

1. Express 2012 as a difference of two integer squares.
2. Each account on a social networking website has a Personal Identification Number, which is required to be a string of eight decimal digits not containing two consecutive digits that are both odd. So for example 31415926 is not an acceptable PIN but 02252012 is an acceptable PIN. Find the number of acceptable PINs.
3. What is the smallest number of integer squares whose sum is 2012?
4. At a small party there are fifteen door prizes, each of which is randomly given to one of the five guests. What is the probability that each of the five guests receives at least one door prize?
5. A circle C_1 has center (h, k) with $h > k > 0$. It is tangent to the x -axis and to the line L , which has equation $y = mx$ for some positive constant m . A smaller circle C_2 is tangent to C_1 , L , and the x -axis. If an infinite family C_1, C_2, C_3, \dots of progressively smaller circles is constructed so that C_{n+1} is tangent to C_n , L , and the x -axis for every n , find the sum of the areas of all the circles in this family in terms of h and k .