College of Charleston
Math Meet 2012
Written Test – Level 2

1. How many digits are in the number $2^{120} \cdot 5^{125}$?
   (A) 114 (B) 124 (C) 126 (D) 141 (E) 142

2. A dish contains twenty wrapped chocolate candies. Eight of them are *dark* chocolate and twelve are *milk* chocolate. Ten of them have *yellow* wrappers and ten have *blue* wrappers. Only two of the candies with yellow wrappers are dark.
   I have randomly selected a candy from the dish and notice that it has a blue wrapper. What is the probability that it is *dark* chocolate?
   (A) $.2$ (B) $.5$ (C) $.6$ (D) $.75$ (E) $.8$

3. How many ordered triples (x, y, z) are solutions to the equation $x + y + z = 9$ if x, y and z are positive integers?
   (A) 28 (B) 30 (C) 32 (D) 34 (E) 36

4. An aquarium has a rectangular base that measures 20 inches by 10 inches. What's the diameter of the smallest circular table that will hold the aquarium, if no part of the base can extend over the edge of the table?
   (A) 10 inches (B) 20 inches (C) 30 inches (D) $7\sqrt{7}$ inches (E) $10\sqrt{5}$ inches

5. Two cards are selected at random from a stack of four cards. We know that the cards in the stack are labeled "1", "2", "3" and "6". What is the probability that the *product* of the selected numbers is 6?
   (A) 0 (B) $1/3$ (C) $2/3$ (D) $1/4$ (E) $1/2$

6. How many seconds are there in exactly six weeks?
   (A) $8!$ (B) $9!$ (C) $10!$ (D) $11!$ (E) $12!$

7. Which geometric description best fits the set of points S?

$$S = \{(x, y) \mid x \geq 0,\ y \geq 0,\ x + 4y \leq 500\}$$

   (A) a connected set of line segments

   (B) a circular region

   (C) a triangular region

   (D) a square region

   (E) a rectangular region with two short and two long sides

8. Express the domain of the function $y = \ln(\sqrt{(x-1)/(x+3)})$ using interval notation.
   (A) $(-\infty, -3) \cup (-3, \infty)$ (B) $(-3, -1)$
   (C) $(-\infty, -3) \cup [1, \infty)$ (D) $(-\infty, -3) \cup (1, \infty)$
   (E) $(-3, -1]$

9. There are six positive integers that can evenly divide the number 2012. They are $1, 2, 4, 503,$ 1006 and 2012 itself. I have written these six numbers on the faces of a regular six-sided die. If I roll the die two times, what is the probability that the sum of the rolls is a divisor of 2012?
   (A) $1/36$ (B) $1/9$ (C) $1/6$ (D) $1/3$ (E) $1/2$

10. Define the sequence $\{a_0, a_1, a_2, \dots\}$ recursively by the formula

$$a_n = \frac{1 + a_{n-1}a_{n-2}}{6}$$

where $a_0 = 7$ and $a_1 = 2$. What is a_3?

(A) 1 (B) $\dfrac{5}{2}$ (C) $\dfrac{7}{3}$ (D) $\dfrac{11}{6}$ (E) 2

11. How many functions of the form $f(x) = mx + b$ satisfy $f(f(x)) = x$?
(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

12. Which of these is a mathematically accurate description of the significance of the formula $\dfrac{a + b - |a - b|}{2}$?

(A) It is always equal to either a or b (whichever is *larger*).

(B) It is always equal to either a or b (whichever is *smaller*).

(C) It either has the value $a - b$ or $b - a$ (whichever is *larger*).

(D) It either has the value $a - b$ or $b - a$ (whichever is *smaller*).

(E) None of the above.

13. There are thirty-two combinations of how five voters can vote either YES or NO on an issue. In how many combinations is the vote either 2 YES to 3 NO or 3 YES to 2 NO?
(A) 6 (B) 10 (C) 20 (D) 26 (E) none of these

14. For any integer n such that $10 \le n \le 99$, define $f(n)$ to be the sum of the digits of n and let

$$g(n) = \frac{n - f(n)}{3}.$$

Which of these are true statements about the range of the function g?

    I. The range of g includes fractions which are not whole numbers.
    II. The average of all of the elements of the range of g is 16.5.
    III. Every element of the range of g is an odd number.

(A) Only I is true.   (B) Only II is true.

(C) Only III is true.   (D) Only II and III are true.

    (E) None of these statements is true.

15. If $-1 < x < y < 0 < u < 1 < v$, which of the following could be true?
(A) $yu = x$ (B) $x = y/v$ (C) $\ln x + \ln y = \ln v.$

    (D) $u + \ln v = v$ (E) $\ln |x| = x$

16. An athlete travels from point A directly to point B. If he runs $3/4$ of the way, walks $1/9$ of the way, and jogs the last 10 miles, what is the distance from point A to point B?
(A) 36 miles   (B) 45 miles   (C) 56 miles   (D) 72 miles   (E) none of these

17. Peter has two thirds as many marbles as Henry. If Peter gets eight more marbles, he will have five sixths as many marbles as Henry. How many marbles do Peter and Henry have altogether?
(A) 60   (B) 65   (C) 70   (D) 75   (E) 80

18. A company sells a liquid used by scientists, but the more they charge for it the less they will sell. If they charge x dollars per gallon for the stuff, they will sell $(2727 - 18x)$ gallons of it. Their revenue (the total amount of money they receive in payment) would then be the product of the price charged per gallon and the number of gallons sold. What price would make the revenue as large as possible?

    (A) $20 per gallon   (B) $36.25 per gallon   (C) $66 per gallon

          (D) $75.75 per gallon   (E) none of the above

19. In a race of 1000 meters, **A** beats **B** by 200 meters and **A** beats **C** by 500 meters. Assuming that the contestants run at constant speeds, by how many meters does **B** beat **C**?

    (A) 300   (B) 375   (C) 450   (D) 500   (E) 625

20. A boat is anchored in the middle of a still lake when its fuel tank begins to leak, causing a circular slick on the surface of the lake. If the radius of the slick is growing at a constant rate of 2 ft/min, approximately how much area will the slick cover after 1 hour?

    (A) $(120)^2\pi$ square feet   (B) $(60)^2\pi$ square feet   (C) $(120)\pi$ square feet

          (D) $(240)^2\pi$ square feet   (E) 240π square feet

21. What is the smallest positive integer m for which there is an integer n such that $2010m + 2012n = 2$?

    (A) 503   (B) 1005   (C) 2010   (D) 2011   (E) none of these

22. The figure to the right can be completed to be a *Magic Square*. This means that you can fill in the empty boxes so that the sums of the entries of every row, every column and both diagonals are all equal. If the box is completed in this way, what entry goes in the *middle* box?

17		
2		
23	8	

    (A) 2   (B) 3   (C) 7   (D) 11   (E) 14

23. Express the area of the trapezoid in terms of the angle θ.
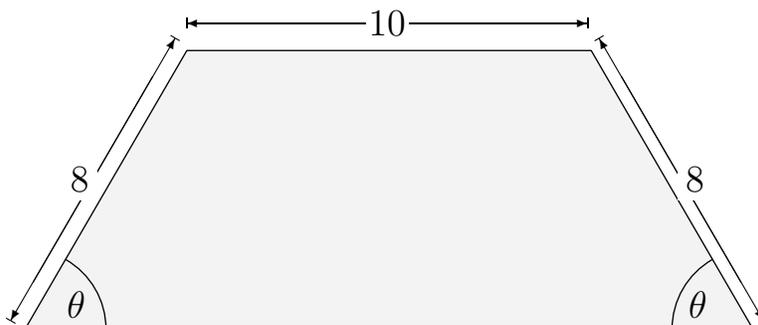
    (A) $4(20 + 2\tan\theta)$ (B) $80\cos\theta + 64\cos\theta\sin\theta$

    (C) $4\sin\theta(10 + 2\cos\theta)$ (D) $4\tan\theta(20 + 16\cot\theta)$

          (E) $80\sin\theta + 32\sin 2\theta$

24. How many real solutions are there to the equation $\sqrt{4x^2 - 1} - \sqrt{2x^2 + 3} = 1$?
    (A) none   (B) 1 (C) 2 (D) 3 (E) more than 3

25. Circle C_1 has radius 2 and is centered at the point $(0, 0)$. Circle C_2 has radius 2 and is centered at the point $(0, 2)$. Find the distance between the points of intersection of C_1 and C_2.
    (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $2\sqrt{2}$ (D) $3\sqrt{2}$ (E) None of these

2012 Answers / Level 2 Test

1. B	10. A	19. B
2. C	11. E	20. A
3. A	12. B	21. B
4. E	13. C	22. E
5. B	14. E	23. E
6. C	15. E	24. C
7. C	16. D	25. B
8. D	17. E	
9. B	18. D	