College of Charleston
Math Meet 2012
Written Test – Level 3

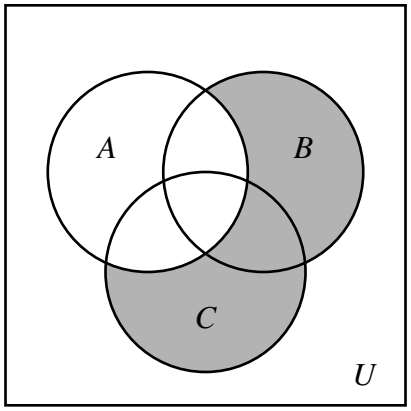
1. Find a_2, where

$$\left(x - \frac{1}{x}\right)^8 = \sum_{k=-8}^{8} a_k x^k$$

(A) +28 (B) -56 (C) +112 (D) -70 (E) +70

2. If A, B, and C are subsets of the set U, then
   - $A \cap B$ is the set of all elements that belong to both A **and** B,
   - $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
   - A^c is the set of all elements of U that are **not** in A.

In the diagram, A, B and C are represented by circles and U is represented by a square.
Which of the following expressions describes the region shaded in the diagram?



(A) $(A \cap B)^c \cup C$ (B) $A^c \cap (B \cup C)$ (C) $(A \cup B^c) \cap C$
(D) $A^c \cup (B \cap C)$ (E) $(A \cup B) \cup C^c$

3. Identify the graph of $y = -3\sin(2x - \pi)$.

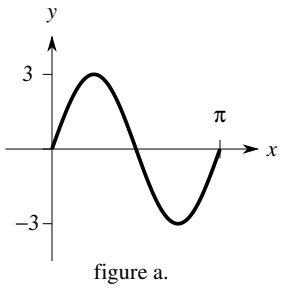


figure a.

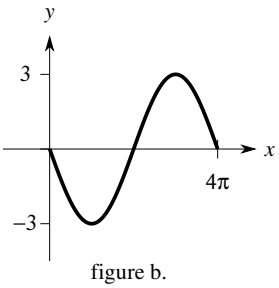


figure b.

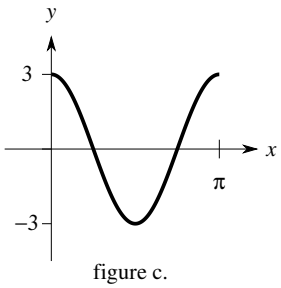


figure c.

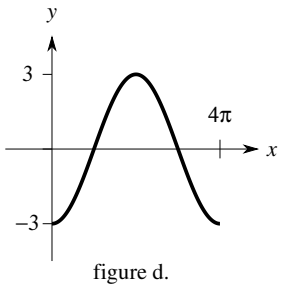


figure d.

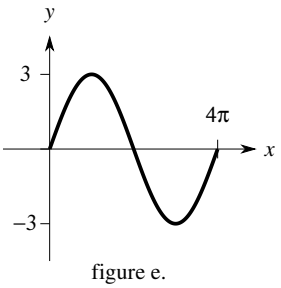


figure e.

(A) figure a. (B) figure b. (C) figure c. (D) figure d. (E) figure e.

4. Farmer Jane keeps four kinds of animals: cows, chickens, ducks, and pigs. Next year, she would like to have 14 cows and chickens (that is, her cows and chickens should total 14), 12 chickens and ducks, 10 ducks and pigs, and 8 pigs and cows. What's the total number of animals must she have altogether?

(A) 22

(B) 32

(C) 42

(D) It's impossible for her to do this.

(E) It's possible for her to do this, but there's more than one possible total.

5. Levin and Alex are playing the following coin-toss game. They take turns tossing the coin, beginning with Levin, and the first player to get tails loses the game.
Assuming that the two outcomes of a coin toss—heads and tails—occur with equal probability, what's the probability that Levin will win the game?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) $\frac{4}{7}$

6. Find the smallest possible degree of a polynomial $p(x)$ that satisfies all of the following.
   - When $p(x)$ is divided by $(x-2)(x-3)$, the remainder is 2.
   - When $p(x)$ is divided by $(x-1)(x-3)$, the remainder is $5-x$.
   - When $p(x)$ is divided by $(x-1)(x-2)$, the remainder is $6-2x$.

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

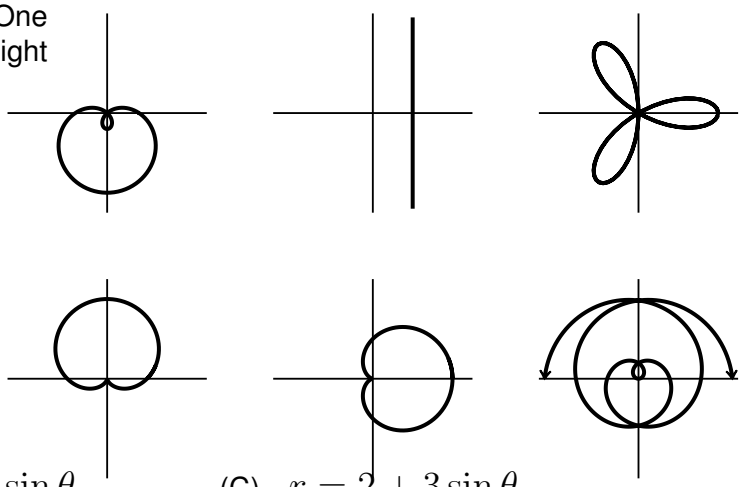
7. Suppose $q(x)$ is a polynomial of degree 5, and that

$$q(1) = q'(1) = q''(1) = q(-1) = q'(-1) = 0.$$

   At what x-value other than ± 1 must $q'(x) = 0$?

   (A) 4/5 (B) -3/5 (C) 2/5 (D) -1/5 (E) 0

8. Find the polar equation whose graph is **not** included in the figure. (Each graph is to a different scale. One unit in one graph and one unit in another graph might not appear equal.)



   (A) $r = \cos(3\theta)$ (B) $r = 1 + \sin\theta$ (C) $r = 2 + 3\sin\theta$
   (D) $r = \sec\theta$ (E) $r = \theta$

9. Which most closely resembles the graph of a function $f(x)$ for which $f'(x) = \sqrt{4-x^2}$? (Graphs not to scale.)

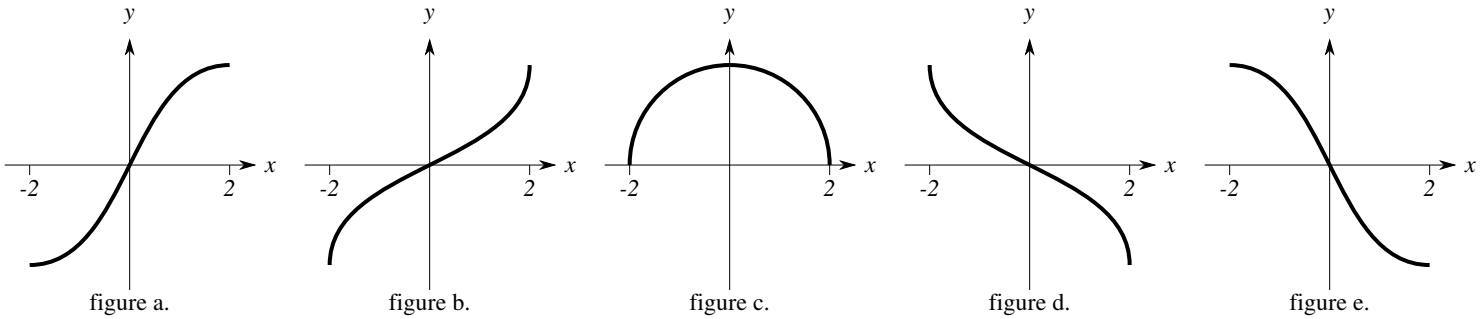


figure a. figure b. figure c. figure d. figure e.

   (A) figure a. (B) figure b. (C) figure c. (D) figure d. (E) figure e.

10. A line segment has endpoints $(0, 0)$ and $(12, a)$. There are exactly four points (possibly including the endpoints) on the line segment which have integer coordinates. Which of the following could be the slope of the line segment?

(A) $-\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) none of these

11. Five candidates—Aay, Bee, Cee, Dea, and Ewa—compete for the Wojcicka Medal in mathematics at the College of Charleston. Each candidate earns between 1 and 5 points in each of five areas—algebra, analysis, topology, discrete math, and logic—and the candidate with the most points wins the medal. There were no ties in any subject, and no ties in overall point totals. Aay came in first with 24 points, Bee came in second, Cee third, Dea fourth, and Ewa fifth. Ewa earned five points in analysis and three points in algebra.
How many points did Cee earn altogether?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

12. The minute hand on a watch is 2 cm long, and the hour hand is 1 cm long. How fast are the tips of these moving together at 4:00? (Answer in cm/minute.)

(A) $\frac{11\pi}{360}$ (B) $\frac{11\pi}{360\sqrt{3}}$ (C) $\frac{11\pi\sqrt{3}}{360\sqrt{7}}$ (D) $\frac{11\pi\sqrt{7}}{360\sqrt{3}}$ (E) $\frac{11\pi}{360\sqrt{7}}$

13. If f is a differentiable function and $g(x) = x^2 f(x)$, then $g'(x) =$

(A) $xf'(x) + f(x)$ (B) $x^2 f'(x) + 2xf(x)$ (C) $x^2 f'(x) + f(x)$
(D) $2x + f''(x)$ (E) $2 + f''(x)$

14. For any integer n such that $10 \leq n \leq 99$, define $f(n)$ to be the sum of the digits of n and let

$$g(n) = \frac{n - f(n)}{3}.$$

Which of these are true statements about the range of the function g?

I. The range of g includes fractions which are not whole numbers.
II. The average of all of the elements of the range of g is 16.5.
III. Every element of the range of g is an odd number.

(A) Only I is true. (B) Only II is true.
(C) Only III is true. (D) Only II and III are true.
(E) None of these statements is true.

15. Compute the fourth derivative of $f(x) = \frac{1 - x^5}{1 - x}$ at $x = 0$.

(A) 4 (B) 5 (C) 15 (D) 24 (E) 120

16. Which of the following equations is true?

I. $5\sqrt{\frac{5}{24}} = \sqrt{5+\frac{5}{24}}$

II. $2\sqrt[3]{\frac{2}{7}} = \sqrt[3]{2+\frac{2}{7}}$

III. $3\sqrt[5]{\frac{2}{4}} = \sqrt[5]{3+\frac{2}{4}}$

(A) None are true (B) Only I is true (C) Only II and III are true

(D) Only I and II are true (E) All three are true

17. A natural number is a palindrome if its digits read the same from left to right as they do from right to left. For example, 3521253 is a palindrome. How many 3-digit numbers are palindromes that are divisible by 11?

(A) none of them are divisible by 11 (B) 4

(C) 8 (D) 22

(E) 90

18. Triangle ABC has area 12. If side AB has length 6 and side BC has length 5, what must be the length of side AC?

(A) 5

(B) $\sqrt{37}$

(C) $\sqrt{97}$

(D) $\sqrt{109}$

(E) More than one answer is possible with the given information.

19. A club with 15 members must choose a delegation of 4 members to serve at a convention. Team members Jacob and Sarah refuse to serve on a delegation together. How many delegations are possible?

(A) 130 (B) 143 (C) 156

(D) 169 (E) None of the above

20. If $\frac{a+13b}{3a-b}$ equals 3, then $\frac{a^3}{b^3}$ equals

(A) 64 (B) 27 (C) 8 (D) $\frac{127}{64}$ (E) 1

21. Simplify $\frac{(2x)^{\ln(x+3)}(x+1)^{\ln(x+2)}}{(x+2)^{\ln(x+1)}(x+3)^{\ln x}}$.

(A) $(x+3)^{\ln 2}$ (B) 2 (C) $\ln 2 \ln(x+3)$

(D) $x^{\ln(x+3)}$ (E) none of these

22. Which of the following is equal to $\dfrac{1}{\sqrt[3]{2}} + \dfrac{1}{\sqrt[6]{2}} + 1 + \sqrt[6]{2} + \sqrt[3]{2} + \sqrt{2}$?

(A) $\dfrac{1}{\sqrt{2} - \sqrt[3]{2}}$ (B) 6 (C) $\dfrac{1}{\sqrt[6]{2} - 1}$ (D) $3\sqrt{2}$ (E) none of these

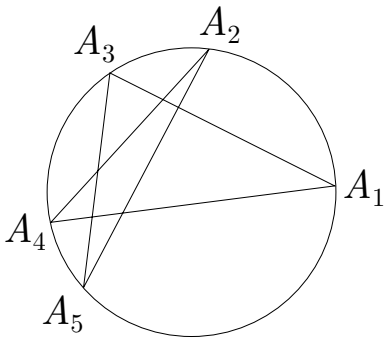
23. The polynomial $x^5 - 40x^2 - 60x - 144$ has a repeated root. Which of the following is a repeated root?

(A) 4 (B) -2 (C) $1 - \sqrt{3}$ (D) $-1 + \sqrt{-5}$ (E) none of these

24. How many real solutions are there to the equation $6^x - 5 \cdot 3^x = 2^{x+2} - 20$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

25. Choose five different points $A_1, \cdots, A_5$ on a circle, arranged in counterclockwise order from A_1 to A_5. Form a star by joining A_1 to A_3 to A_5 to A_2 to A_4 to A_1. In this star, what is the sum of the vertex angles $\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5$?



(A) $\dfrac{2\pi}{5}$

(B) $\dfrac{\pi}{2}$

(C) π

(D) 2π

(E) it depends on the positions of the points.

2012 Answers / Level 3 Test

1. B
2. B
3. A
4. D
5. B
6. B
7. D
8. C
9. A
10. C
11. C
12. C
13. B
14. E
15. D
16. D
17. C
18. E
19. E
20. C
21. A
22. A
23. D
24. C
25. C