C of C Math Marathon 2013

Rules:

i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.

ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.

iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.

iv. The judges' decisions will be final.

v. All papers are to be mailed to the following address or submitted electronically to `mathmeet@cofc.edu`:

Math Meet (Marathon)
Department of Mathematics
College of Charleston
66 George Street
Charleston, SC 29424

vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .

vii. All entries must be received or postmarked by February 12, 2013.

The Questions:

1. What is the smallest possible surface area for a rectangular box with sides of integer length and volume 2013 cubic units?

2. The random number generator on a computer is used to generate two numbers a, b in the interval $[0, 10]$. What is the probability that
$$\log_b(\log_b(\log_b a))$$
is defined?

3. I have a set of squares with integer sides whose average area is 2013. This set consists of as few squares as possible such that the above condition is satisfied; furthermore the average perimeter of these squares is as small as possible such that the above conditions are satisfied. What are the squares in this set?

4. For some non-zero constants a, b, c, and d, the function
$$f(x) = \frac{ax+b}{cx+d}$$
is its own inverse, and the function $g(x) = f(1/x)$ is also its own inverse. If $f(2013) = 2013$, find $f(-1) + f(0) + f(1)$.

5. A game is played on the vertices of the cube $\{(x, y, z) : 0 \leq x, y, z \leq 1\}$ as follows. Beginning at $(0, 0, 0)$, the player moves along one of the edges emanating from that vertex to an adjacent vertex according to a roll of a die; if a 1 or 2 is rolled, she moves in the x-direction; if a 3 or 4 is rolled, in the y-direction; and if a 5 or 6 is rolled, in the z-direction. The player continues to move in this manner, according to the roll of a die, until she reaches either (1,1,0) or (1,1,1). If she reaches (1,1,1), the game is won; if she reaches (1,1,0), the game is lost. What is the probability that she wins the game?