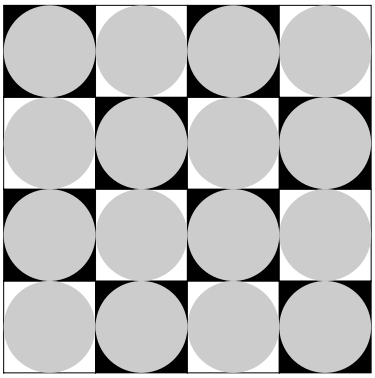
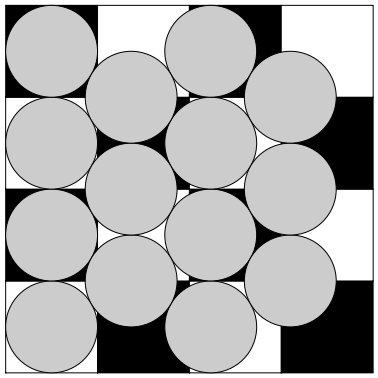
College of Charleston Math Meet 2013
All-Day Sprint: The Niederman Challenge

Derrick Niederman, an adjunct math professor at the College, has a reputation for being able to make mathematical puzzles and games that are very challenging and also very interesting. If you don't believe me, check out his Wikipedia page...or just try the six puzzles he created below just for *you*. We hope you'll agree that they prove the reputation is well-deserved! Put your team's answers in the corresponding boxes on the back. **You do not have to answer them all;** the winner will be selected randomly from among those schools that have the maximum number of correct answers.

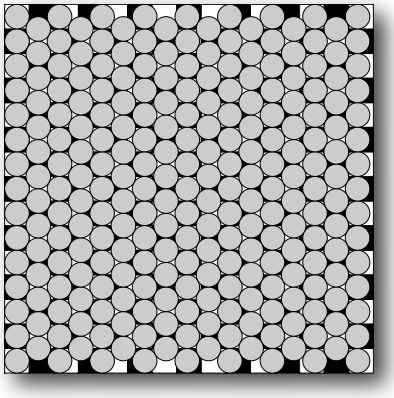
1) Suppose you want to arrange circular disks with 1 inch diameter in a single layer entirely on a "checkerboard" of n rows and columns of 1 inch by 1 inch squares. If you use the standard arrangement in which each disk is centered on a square, you can fit at most n^2 disks. Alternatively, you could use the "hexagonal" arrangement in which the vertical centers of the disks align with the tops and bottoms of those in the preceding column. As this figure shows, you can fit more disks using the standard arrangement than the hexagonal one when n=4, but when n=15 the hexagonal arrangement fits 247 disks onto the board of 225 squares.



Standard, n=4



Hexagonal, n=4



Hexagonal, n=15

Question 1: What is the smallest whole number n for which the hexagonal arrangement fits more disks than the standard arrangement?

2) The operator *Roman* writes a positive integer as a Roman numeral, so, for example, *Roman*(115) = CXV. The operator *Number* takes a string of letters and converts it into the sum of the values of the letter's alphabetical positions (A=1, B=2, C=3,...). For example, *Number*(CXV) equals 3 + 24 + 22 = 49.

Question 2: Find two solutions to the equation *Number*(*Roman*(x)) = x.

3) A man decided to do a little reading before going to bed. As he finished for the evening his wife asked him which pages of the book he had just read. "The sum of the consecutive page numbers I just read was either 412 or 512. It's tough to do the arithmetic in my head, so I'm not sure which number is right."

Question 3: Which pages of the book had he just completed?

4) Consider the function which turns a given positive whole number into the number of letters in the English expression for that number (without the word “and”). For example, 124 becomes 20 because “one hundred twenty-four” has twenty letters. This operation can be *iterated* (repeated) endlessly. As before, 124 becomes 20 and then 20 becomes 6 (because “twenty” has 6 letters), and 6 becomes... The interesting thing is, there is a number *A* such that no matter what number you started with this procedure eventually leads to and *stays at* the number *A.* (*A* stands for “attractor” since that is the technical term for it in the field of dynamical systems.)

   Question 4: What is *A?*

5) The previous question is a little more interesting if the numbers are written out in *Spanish* rather than English. In that case, there is again a number *A* with the property that many starting numbers eventually get turned into and stay forever at the value *A.* However, some starting choices lead to a different longterm behavior that bounces back and forth endlessly between two numbers we will call *B* and C.

   Question 5: What is *A*(B+C)?*

6) The total number of 10 letter strings where each character has to be either an H or a T is obviously 2^{10}. How many are there if you can never have two or more T’s in a row?

Answers:

1.

2.

3.

4.

5.

6.