The $3 \cdot 5^2$-th Number Games had begun. After running through a forest of binary trees, tripping over square roots, and dodging convergent nets, Katenary Evenmean and her friend Pitau arrived at the abandoned cone-u-copia at the center of the field and took a moment to catch their breaths.

“Look,” said Pitau, “there’s a device hidden under the cone-u-copia. That must be the control panel we were told about. Maybe we can break into it and open the spherical boundary.”

Katenary nodded, and used a wedge to pry it open. “It looks like a clock,” she said. “There are instructions...”

**Clock of Doom All-Day Sprint**

Instructions for Opening Clock of Doom Access Panel

- Enter a $+$ or $\cdot$ in each sector of the second ring. There are four $+$ signs and eight $\cdot$ signs. The two $\cdot$ signs in sectors 2 and 5 are fixed and cannot be changed.

- Enter the numbers 1 through 12 in the sectors of the outer ring.

- Each number is used exactly once.

- The product of the two numbers in a sector with a $\cdot$ must be a multiple of 12. The sum of the two numbers in a sector with a $+$ must be a multiple of 12. The three numbers paired with 7, 8, and 9 (shaded area) sum to exactly 12.
After tinkering for a while, Katenary and Pitau managed to set the clock just right, and sure enough, the side of the device opened, revealing another mechanism and more instructions:

**Clock of Doom Boundary Control**

To open the boundary of the arena sphere, enter in the space below all pairs of integers \((a_1, a_2)\) such that

- \(1 \leq a_1 \leq 11\) and \(1 \leq a_2 \leq 11\)
- the polynomial \(f(x) = a_1 x + a_2 x^2\), when interpreted as a function on the integers modulo 12, is a bijection.

“Bijection? Modulo 12?” asked Pitau, “What does all that mean?”

Katenary replied, “The modulo 12 part means that you do integer arithmetic, but as if you’re working on a clock, so you reduce every result to its remainder when divided by 12. For instance, \(8 + 8 \equiv 16 \equiv 4 \pmod{12}\), and \(9 \cdot 4 \equiv 36 \equiv 0 \pmod{12}\). The bijection part means that \(f\) is a one-to-one correspondence. In other words, the output value of \(f\) cannot be equal for two different inputs between 0 and 11.”

“This sounds hard,” Pitau said. “I can see that \(f(0) \equiv 0\) no matter what \(a_1\) and \(a_2\) are, so that’s a start. But is there a way to find these polynomials without checking every possible one? How many are we supposed to find?”

“I don’t know,” she replied. “There’s space to enter several . . .”