

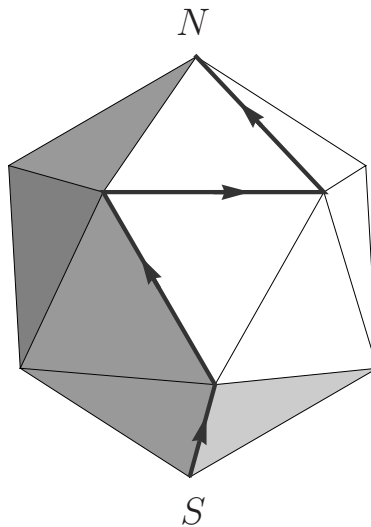
The Icosahedral Cage Challenge

All-day Sprint

College of Charleston Math Meet
February 2014

A giant cage is built in the shape of an icosahedron, a closed surface formed from 20 congruent equilateral triangles as shown in the figure. The cage stands on one of its 12 vertices, called S . To escape from inside the cage, the contestants in The Number Games must climb from S all the way to an exit located at vertex N , the very top of the cage.

Because the triangular faces of the cage are electrified, the contestants can move from S to N *only* by moving along one of the 30 edges. Furthermore, once an edge has been traversed by a contestant, the Game Maker ensures that same contestant cannot follow that same edge again without being electrocuted. So, the contestants can travel only along a *safe edge path*, a sequence of adjacent edges which does not involve repeating an edge (traversed in either direction), although the same vertices can be visited repeatedly.



The length of a safe edge path is the number of edges in the path. For example, the picture above shows a safe edge path of length 4 that starts at S and ends at N .

1. The first contestant can escape using a path of length 3. How many safe edge paths are there that start at S , end at N , and have length exactly 3?
2. The second contestant must take a path of length 4. How many safe edge paths are there that start at S , end at N , and have length exactly 4?
3. All of the rest of the contestants can only escape the cage if they take a path of length 5. How many safe edge paths are there that start at S , end at N , and have length exactly 5?