

C of C Math Marathon 2014

Rules:

- i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- iv. The judges' decisions will be final.
- v. All papers are to be mailed to the following address or submitted electronically to mathmeet@cofc.edu:
Math Meet (Marathon)
Department of Mathematics
College of Charleston
66 George Street
Charleston, SC 29424
- vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .
- vii. All entries must be received or postmarked by February 12, 2014.

The Questions:

1. Find all expressions of 2014 as the sum of a set of consecutive positive integers.
2. A playoff series between the Hawks and the Knights continues until one of the two teams has won three games, and therefore may take as many as five games. The probability that the Hawks defeat the Knights in any given game is the same for each game. The probability that the series takes only three games is equal to the probability that the series takes longer than three games. What is the probability that the series takes five games?
3. A rectangular sheet of paper is folded so that its diagonally opposite corners meet. The fold divides the paper into two halves of equal area, but when folded in this way there are two triangular regions which do not overlap the other half. The ratio of the sum of the areas in these two triangles to the area of the paper is equal to the ratio of width to length of the paper. Find this ratio.
4. An integer n is chosen at random from the set $\{1, 2, \dots, 2014\}$ and an integer k is chosen at random from the set $\{0, 1, 2\}$. Then a deck of $3n$ cards labeled $\{1, 1, 1, 2, 2, 2, \dots, n, n, n\}$ is produced, and a card is randomly selected and removed from this deck. At this point k cards labeled 0 are added to the deck. A second card is then randomly selected from this deck and removed. Finally a third card is randomly selected from the deck and removed. What is the probability that all three cards which were selected and removed are labeled with the same number?
5. A particle begins at a point in the Cartesian plane and travels horizontally to the right for a distance of one unit. From there it moves up and to the right along a line of positive slope m for a distance of one unit. From there it returns to the original starting point along a line of positive slope M . Express M as an algebraic function of m .