The Number Games Sprint

College of Charleston Math Meet
February 22, 2014

Instructions: Write the exact answer to each question in the corresponding box. Remember that the winners in this event are those participants who answer the most questions correctly *in a row* beginning with the first question. So, try to get as far as you can without making a mistake!

1. Suppose x and y are nonzero real numbers satisfying $x + y = 0$. Find the exact value of $\dfrac{x^{2014}}{y^{2014}}$.

    # 1:

2. What are the last two digits of 11^{2014}?

    # 2:

3. Suppose x and y are two real numbers such that $x - y = 3$ and $x^2 + y^2 = 4037$. Find the value of xy.

    # 3:

4. Suppose x and y are two real numbers such that $\sqrt{x} + \sqrt{y} = 50$ and $x + y = 2014$. Find the value of $\sqrt{xy}$.

    # 4:

5. Every integer has a unique prime factorization. For example, the prime factorization of 60 is $2^2 \times 3 \times 5$ and the prime factorization of 27 is 3^3. Let $p(x) = (x + 3)^{1007} = c_0 + c_1 x + c_2 x^2 + \cdots + c_{1007} x^{1007}$ and let $N = c_0 + c_1 + c_2 + \cdots + c_{1006} + c_{1007}$ be the sum of the coefficients from its expanded form. What is the prime factorization of N?

    # 5:

6. Recall that $n! = n(n-1)(n-2)\cdots(3)(2)(1)$ is the *factorial* of n. Determine the remainder when $(1! + 2! + \cdots + 2014!)^2$ is divided by 4.

    # 6:

7. $2014!$ ends in a lot of zeros. How many are there?

    # 7:

8. Express the following product as a fraction in lowest terms:

$$2 \cdot \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{2014^2}\right)$$

    # 8:

Math Meet
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Timed Sprint

Name (please print): _____

School Name: _____

Grade: _____

The grading for the Timed Sprints is unusual! Your grade will be the number of questions answered correctly, starting with the first question, before you make a mistake. For example, if you only answer questions 1-4 correctly and questions 7-13 correctly, your grade will be a "4" since you did not get question 5 right. You will have a limited amount of time to work on the sprint. Your paper will be collected at the end of this period.

By my signature below I certify that all of the work completed on this sprint is my own.

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