COLLEGE of CHARLESTON

MATH MEET 2018

Number Piles

The preferred number system among numberphiles is the Number Pile system. Number Pile uses six primary symbols to denote the numbers zero through five. One can form a compound symbol by stacking two primary symbols on top of each other $\begin{smallmatrix} x \\ y \end{smallmatrix}$, as long as $x > y$. One can also form a compound symbol by stacking a row of two primary symbols on top of a third primary symbol $\begin{smallmatrix} x\,y \\ z \end{smallmatrix}$, as long as $x > y > z$. The numerical value of a compound symbol is the sum of the values of the primary symbols it contains.

Primary Symbol	Meaning
◯	0
−	1
‖	2
△	3
♦	4
★	5

For example, the compound symbol $\begin{smallmatrix} \blacklozenge \\ - \end{smallmatrix}$ has the value $4+1=5$ and the compound symbol $\begin{smallmatrix} \bigstar\blacklozenge \\ \| \end{smallmatrix}$ has the value $5+4+2=11$. On the other hand, neither $\begin{smallmatrix} \triangle \\ \bigstar \end{smallmatrix}$ nor $\begin{smallmatrix} \|\bigcirc \\ \bigcirc \end{smallmatrix}$ is a legitimate compound symbol, as it's not true that $3>5$ and it's not true that $2>0>0$.

In the usual Arabic numerals, we write numbers using a string of digits, $d_n d_{n-1} \ldots d_1 d_0$, with each digit multiplied by a power of ten, and then the results summed.

$$d_n d_{n-1} \ldots d_1 d_0 = d_n \cdot 10^n + d_{n-1} \cdot 10^{n-1} + \cdots + d_1 \cdot 10^1 + d_0 \cdot 10^0$$

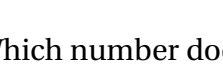
Similarly, we can write Number Pile numbers using a string of compound or primary symbols

$$c_n c_{n-1} \ldots c_1 c_0 = c_n \cdot 10^n + c_{n-1} \cdot 10^{n-1} + \cdots + c_1 \cdot 10^1 + c_0 \cdot 10^0$$

For example, $\bigstar \begin{smallmatrix} \triangle- \\ \bigcirc \end{smallmatrix} \begin{smallmatrix} \blacklozenge\| \\ - \end{smallmatrix} = 5 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0 = 547$.

However, there's a complication compared to the usual number system. There can be several different ways to write the same number. One cause of this is that different compound symbols can represent the same number in Number Pile. Both $\begin{smallmatrix} \blacklozenge \\ - \end{smallmatrix}$ and $\begin{smallmatrix} \triangle \\ \| \end{smallmatrix}$ represent the number 5, and there are several other compound symbols for 5, and also the primary symbol ★. A second reason that numbers can be written in multiple ways is that compound and primary symbols represent the numbers 0 through 12, rather than 0 through 9. For example $\begin{smallmatrix} \triangle \\ - \end{smallmatrix} \begin{smallmatrix} \bigstar\blacklozenge \\ \| \end{smallmatrix} = 4 \cdot 10^1 + 11 \cdot 10^0 = 51$ and $\begin{smallmatrix} \triangle \\ \| \end{smallmatrix} \begin{smallmatrix} - \\ \bigcirc \end{smallmatrix} = 5 \cdot 10^1 + 1 \cdot 10^0 = 51$ both represent the number 51.

Answer the following questions. Write all of your answers using the usual Arabic numerals, not Number Pile.

1. Which number does  represent?

   906

2. Which number does represent?

   1001

3. In how many different ways could we write the number 7 in Number Pile?

   5

4. In how many different ways could we write the number 86 in Number Pile?

   20

5. In how many different ways could we write the number 72 in Number Pile?

   15

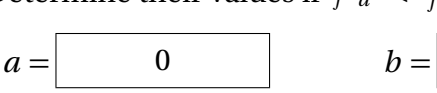
6. Divide  by ||.

   2202364553123

7. What are x and y if  equals ?

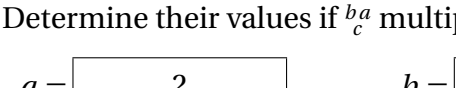
   $x =$ 3

   $y =$ 4

8. Each of a, b, c, d, e, and f is a different primary symbol. Determine their values if  .

   $a =$ 0 $b =$ 4 $c =$ 2

   $d =$ 5 $e =$ 3 $f =$ 1

9. Each of a, b, c, d, e, and f is a different primary symbol. Determine their values if  .

   $a =$ 2 $b =$ 4 $c =$ 1

   $d =$ 0 $e =$ 5 $f =$ 3