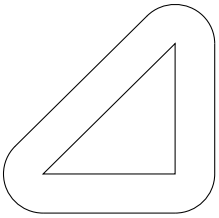
College of Charleston
Math Meet 2014
Written Test – Level 1

1. A fort is in the shape of an isosceles right triangle, with legs of length 10m. There is a moat surrounding the fort, extending 3m out in every direction, as shown in the figure. What is the area of the moat?



(A) 78m^2 (B) 120m^2 (C) $(9\pi + 60 + 30\sqrt{2})\text{m}^2$
(D) $(6\pi + 20 + 10\sqrt{2})\text{m}^2$ (E) none of these

2. A triangle with area 60 has two sides of lengths x and y that form an angle of 15 degrees. Another triangle has sides of length $2x$ and $y/3$ which form an angle of 165 degrees. What is the area of the second triangle?

(A) $15\sqrt{2}$ (B) 20
(C) 40 (D) $60/\sqrt{3}$
(E) Not enough information to answer

3. If $3a + 4a + 6a = \dfrac{b}{2} + \dfrac{b}{3} + \dfrac{b}{4}$ and $a \neq 0$, what is b/a?

(A) $\dfrac{1}{12}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{13}{12}$ (D) 6 (E) 12

4. A group of good friends go out to dinner at a restaurant and agree that they will split the cost evenly. However, by the time the bill totaling $\$87.50$ has arrived, two of them have left without paying. (Apparently, they were not really such good "friends" after all.) The remaining dinner guests are a little angry, but realize they only have to pay an extra $\$5$ each to cover the cost. How many people were in the original group?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

5. What is the average of the four *prime* numbers closest to 58?

(A) 57 (B) 58 (C) 59 (D) 60 (E) 61

6. Let x be the solution to $\sqrt{x - \sqrt{26 + x}} + \sqrt{x + \sqrt{26 + x}} = 6$. Which of these statements is true?

(A) $x - 1$ is a prime number (B) $x - 2$ is a prime number (C) $x - 3$ is a prime number
(D) $x > 14$ (E) $x^2 < 87$

7. Which choice is closest to $\sqrt{410100200300}$?

(A) 6.03×10^4 (B) 2.03×10^5 (C) 6.40×10^4 (D) 2.03×10^4 (E) 6.40×10^5

8. A student who believed that common digits could be cancelled from the numerator and denominator of a fraction would think that

$$\frac{23}{35} = \frac{2}{5}.$$

Of course, this is *not* correct. For which of the following fractions would that misguided logic, just by coincidence, produce the correct result?

(A) $\frac{24}{45}$ (B) $\frac{25}{55}$ (C) $\frac{26}{65}$ (D) $\frac{27}{75}$ (E) $\frac{28}{85}$

9. Two four-digit numbers $abcd$ exist such that

$$abcd + a + b + c = 2014.$$

What is the product of the digits in the smaller of the two?

(A) 0 (B) 2 (C) 288 (D) 324 (E) 405

10. Let C be the number of gallons in a cubic foot. If a spherical fish tank has volume 36 gallons, what is its radius (in feet)?

(A) $\sqrt[3]{\frac{3}{\pi C}}$ (B) $\frac{3}{C\sqrt[3]{\pi}}$ (C) $\frac{3C}{\sqrt[3]{\pi}}$

(D) $3\sqrt[3]{\frac{C}{\pi}}$ (E) none of these

11. What is the remainder when the polynomial $202x^2 + 2014$ is divided by $x - 10$?

(A) -22014 (B) 20214 (C) 22101 (D) 22214 (E) 241022

12. Find the number of points of intersection between the two curves $1 - 6xy + 3y^2 = 0$ and $1 + 6xy + 3x^2 = 0$.

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

13. The course grade for students in Ms. Noether's math class is based on the sum of the three 100 point tests they take. One student had to miss a test for a good reason. Ms. Noether is considering two schemes to correct this problem in a fair way. Under Scheme A, she will pretend that the student's score on the missing test was the average of the other two scores. Under Scheme B, she will multiply the sum of the two scores from tests taken by $300/200$ to "scale it up" so that it is a score out of 300 points. Under which scheme will the student have a higher grade?

(A) Scheme A always gives a higher grade.

(B) Scheme B always gives a higher grade.

(C) Scheme A if the average of the two tests taken is above $66.\overline{66}$.

(D) Scheme A if the average of the two tests taken is below $66.\overline{66}$.

(E) The two schemes always give the same value.

14. Let S denote the set of all five-digit numbers in which the sum of the digits is equal to 43. Let S' be the subset of S of elements which are divisible by 11. What is the ratio of the size of the set S' to the size of the set S?

(A) $1/3$ (B) $1/5$ (C) $1/11$ (D) $1/15$ (E) 0

15. A standard deck containing 52 cards consists of 13 different "kinds" of cards (Ace, 2, 3, ..., 10, Jack, Queen, and King), each in four different "suits" (hearts, diamonds, clubs, and spades). Three cards are randomly selected (without replacement) from a standard deck. What is the probability that all three are of the same kind?

(A) $\frac{6}{2550}$ (B) $\frac{12}{2652}$ (C) $\frac{24}{132,600}$ (D) $\frac{24}{2704}$ (E) $\frac{12}{2550}$

16. If $\frac{13}{40} = \frac{1}{a} + \frac{1}{b}$ where a and b are positive integers, what is $a^2 + b^2$?

(A) 37 (B) 45 (C) 53 (D) 74 (E) 89

17. Spinner A has a probability of 0.6 of landing on green and 0.4 of landing on red. Spinner B has probability 0.2 of landing on green and 0.8 of landing on red. What is the probability that they land on the same color?

(A) 0.44 (B) 0.50 (C) 0.56 (D) 0.60 (E) 0.64

18. We have seven coins: three identical pennies, two identical nickels, a dime, and a quarter. How many ways are there to line up these coins (if rearranging the identical pennies and nickels aren't counted)?

(A) 5041 (B) 720 (C) 420 (D) 49

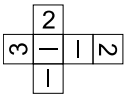
19. What is a_{2014} if $a_1 = 4$, $a_2 = 3$ and $a_n = |a_{n-1} - a_{n-2}|$ for $n > 2$?

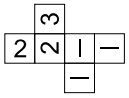
(A) 13 (B) 0 (C) 8 (D) 1 (E) 2

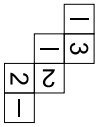
20. Professor Gollub never lies except on one day of the week (always the same) during which he lies all the time. On how many days of the week can Professor Gollub say *If I did not lie yesterday, I will certainly lie tomorrow*?

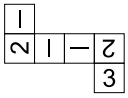
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

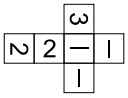
21. The shape in the figure to the right can be folded up to form a cube. Which of the following shapes can be folded up to form a cube that is congruent to it?

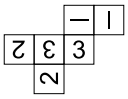


(A) 

(B) 

(C) 

(D) 

(E) 

22. After $\frac{x^2 + x - 6}{x^2 + 2x - 3}$ is reduced to lowest terms, the sum of the numerator and denominator is

(A) $2x^2 + 3x - 9$ (B) $2x - 2$ (C) $2x + 1$

(D) $2x - 3$ (E) 6

23. Given below are three sets of points.

I. $\{(-1, 0), (-2, 1), (-1, 3)\}$
II. $\{(3, -6), (3, -3), (4, -3)\}$
III. $\{(1, 1), (4, 3), (-3, 7)\}$

Which, if any of these sets, represent the vertices of a right triangle?

(A) I only (B) II only (C) II and III only

(D) I and II and III (E) none of these

24. How many three-digit numbers are perfect squares, but not perfect cubes?

(A) 19 (B) 20 (C) 21 (D) 22 (E) none of these

25. Define an operation $*$ on the non-negative integers as follows:
$0 * n = 2n$;
$m * 0 = 0$ if $m \geq 1$;
$m * 1 = 2$ if $m \geq 1$;
$m * n = (m - 1) * (m * (n - 1))$ if $m \geq 1$ and $n \geq 2$.
Find $2 * 2$.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

2014 Answers / Level 1 Test

1. C
2. C
3. E
4. A
5. D
6. C
7. E
8. C
9. E
10. E
11. D
12. E
13. E
14. B
15. A
16. E
17. A
18. C
19. B
20. D
21. C
22. D
23. C
24. C
25. C