Do not open this test booklet until instructed to do so by the proctor. Please turn off and put away all calculators, cell phones and other electronic devices. Their use is not allowed during the test.

This is the Level 1 Written Test. If that is not the level you should be taking, speak to the proctor now to find out where you should be.

You will need three numbers to properly identify yourself on the answer sheet. One is your test booklet number, 5871. No other student has the same test number. The other two numbers you need to know are your six digit Math Meet ID Number and your six digit School Code which your teacher should have provided you. If you do not know them, contact the proctor now for assistance. If you take this test without correctly providing these three numbers then your score may not be correct, you may not be eligible for winning prizes, and you may not be able to obtain your score at all.

Before the test starts, fill in the following information on the answer sheet.

- Write your name (your lastname, then a blank space, then your firstname) in the space provided at the top left. Be sure to write the letters in the boxes and also fill in the corresponding circles below.
- Put your six digit Math Meet ID number in columns A–F, the Test Booklet Number (5871) in columns G–J, and your six digit school code in columns K–P, as shown below:

<table>
<thead>
<tr>
<th>IDENTIFICATION NUMBER</th>
<th>SPECIAL CODES</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C D E F G H I J</td>
<td>K L M N O P</td>
</tr>
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- You do not need to give us your birthdate, grade or sex.

Be sure to completely fill the circles and avoid making any stray marks on the answer sheet.

You will have 60 minutes to answer the 25 multiple choice questions in this booklet. Some of these questions are very hard. Do not be discouraged if you are not able to answer all of them in the time allotted.

The questions are printed in a random order. This means that they are not the same as the order of the questions in other test booklets and they are not ordered so that the easiest ones are at the start.

**Grading Policy:** You will receive 5 points for each correct answer, 1 point for each question which you do not answer, and 0 points for an incorrect answer. (Note that you get more points for not answering than for an incorrect answer.) If two students have the same score, the winner of the tie will be the student who has fewer incorrect answers.
Use this page for “scratchwork”
Use this page for “scratchwork”
1. A pyramid has a rectangular base with area 100 square inches, and its other four faces are equilateral triangles. What is the height, in inches, of the pyramid?
(A) $5\sqrt{2}$  (B) $5\sqrt{3}$  (C) 10  (D) $10\sqrt{2}$  (E) $10\sqrt{3}$

2. Find the sum of all real roots of $x^4 - x^2 - 20 = 0$.
(A) 0  (B) 1  (C) 4  (D) 5  (E) 9

3. Levin and Alex are playing the following coin-toss game. They take turns tossing the coin, beginning with Levin, and the first player to get tails loses the game. Assuming that the two outcomes of a coin toss—heads and tails—occur with equal probability, what’s the probability that Levin will win the game?
(A) $\frac{1}{2}$  (B) $\frac{1}{3}$  (C) $\frac{1}{4}$  (D) $\frac{2}{5}$  (E) $\frac{4}{7}$

4. A two digit integer is $x$ times the sum of its digits. The integer formed by reversing the digits of the original number is the sum of the digits of the original number multiplied by what?
(A) $x$  (B) $11 - x$  (C) $9 - 2x$  (D) $\frac{1}{x}$  (E) none of these

5. The three sides of a triangle have lengths 2, 3, and 4. Find the area of the triangle.
(A) $\frac{\sqrt{90}}{10}$
(B) $\frac{3\sqrt{15}}{4}$
(C) $\frac{9}{2}$
(D) $3\sqrt{3}$
(E) Not enough information is given to determine the area.

6. What is the value of the sum $0 - 1 + 2 - 3 + 4 - 5 + \cdots + 2016$?
(A) $-1$  (B) 0  (C) 504  (D) 1008  (E) 1512

7. Right now, there is a room here at the College where nineteen students are taking this test. Eleven of them are from South Carolina. Ten of the students are female. Only two of the male students in the room are from outside South Carolina. How many of the students in the room are female and from South Carolina?
(A) 0  (B) 1  (C) 2  (D) 3  (E) 4
8. Three circles pass through the origin of a Cartesian plane. The center of the first circle belongs to the first quadrant, the center of the second circle belongs to the second quadrant, and the center of the third circle belongs to the third quadrant. Let $P$ be the intersection of the interiors of the three circles. Find the true statement.
   (A) $P$ must be the empty set.
   (B) $P$ may be nonempty, in which case it must be a subset of quadrant one.
   (C) $P$ may be nonempty, in which case it must be a subset of quadrant two.
   (D) $P$ may be nonempty, in which case it must be a subset of quadrant one or three.
   (E) $P$ may be nonempty, in which case it must be a subset of quadrant four.

9. \[ x = 0, \ y = 0, \ z = 18 \] is one integer solution to \[ xyz - 3xy - 2xz - yz + 6x + 2z + 3y = 36. \] How many integer solutions are there altogether?
   (A) 27  \hspace{1cm} (B) 36  \hspace{1cm} (C) 94  \hspace{1cm} (D) 96  \hspace{1cm} (E) 108

10. Suppose $x$ is a positive whole number which has $n$ zeros in its base two representation. Which of these things must be true about the base two representation of $2x$?
   (A) It has $n$ ones. \hspace{1cm} (B) It has $n - 1$ zeros. \hspace{1cm} (C) It also has $n$ zeros.
   (D) It has $n + 1$ ones. \hspace{1cm} (E) It has $n + 1$ zeros.

11. The ancient Mayans were unusual in that they used a "vegesimal" number system. This means that it was very much like our number system, but base-twenty rather than base-ten. They also wrote their digits in a vertical column and their symbol for zero looked sort of like an empty basket (⊖).

Which of these choices was their way of writing the number 2005?
   (A) .. ⊖ .. \hspace{1cm} (B) ⊖ .. \hspace{1cm} (C) ⊖ ⊖ ⊖ \hspace{1cm} (D) ⊖ ⊖ ⊖ \hspace{1cm} (E) ⊖ ⊖ ..

12. A tank in the shape of an inverted cone is partially filled with water. When more water is added, the height of the water in the tank increases by 10 cm, and the volume of water in the tank doubles. Find the original height (in cm) of water in the tank.
   \[
   \begin{align*}
   & (A) \quad \frac{10}{\sqrt{2} - 1} \quad \quad (B) \quad \frac{10}{\sqrt{2}} \quad (C) \quad 5\sqrt[3]{4} \\
   & (D) \quad \frac{5}{\sqrt{2} + 1} \quad \quad (E) \quad \text{Not enough information.}
   \end{align*}
   \]

13. How many three-digit numbers are divisible by their final (rightmost) digit?
   (A) 100 \hspace{1cm} (B) 319 \hspace{1cm} (C) 329
   (D) 420 \hspace{1cm} (E) none of these

14. One person computes the average of a list of 100 test scores and writes it at the end of the list. Call this number $a_1$. Another person finds the average of those 101 numbers (the test scores and their average $a_1$). Call this second average $a_2$. If for those 100 test scores, the average $a_1$ was less than the median score $m$, what must be true?
   (A) $a_1 > a_2$ \hspace{1cm} (B) $a_2 > a_1$ \hspace{1cm} (C) $a_1 = a_2$
   (D) $m < a_2$ \hspace{1cm} (E) none of the above
15. $x$, $y$, and $z$ are nonzero numbers. If $x$ is 30% of $z$ and $y$ is 6% of $z$, then $y$ is what percent of $x$?
(A) 5%  
(B) 15%  
(C) 18%  
(D) 20%  
(E) none of these

16. Segment $RS$ is a diameter of a circle with center $O$. If $T$ is a point on the circle such that angle $T RS$ measures 50 degrees, find the measure of angle $OTS$.
(A) 40 degrees  
(B) 50 degrees  
(C) 60 degrees  
(D) 90 degrees  
(E) none of these

17. Three circles of radius $r$ are tangent to one another and are circumscribed by a circle of radius $R$. Which statement is true?
(A) $r = \frac{R}{3}$  
(B) $r = \frac{R}{2}$  
(C) $r = R(2\sqrt{3} - 3)$  
(D) $r = \frac{R\sqrt{3} - 1}{2}$  
(E) None of the above

18. In a survey concerning a local issue, 19 percent of doctors voted YES and 95 percent of lawyers voted YES. There is no one who is both a doctor and a lawyer. If all doctors and lawyers voted and 91 percent of all doctors and lawyers voted YES, what is the lawyer to doctor ratio?
(A) 91 to 19  
(B) 95 to 91  
(C) 18 to 1  
(D) 36 to 1  
(E) none of these

19. A recent poll of lowcountry students revealed the following: 8 liked pluff mud but not oysters, 11 liked boiled peanuts but not oysters, 9 liked boiled peanuts but not pluff mud, 4 liked oysters but not pluff mud, and 7 liked pluff mud but not boiled peanuts. How many students liked oysters but not boiled peanuts?
(A) 5  
(B) 6  
(C) 8  
(D) 9  
(E) Not enough information to tell

20. For any positive integer $n$, the function $\tau(n)$ is defined to be the number of factors of $n$. For example, $\tau(10) = 4$ because 10 has 4 factors: 1, 2, 5, and 10. What must be true if $\tau(m)$ is an odd number?
(A) $m$ is odd  
(B) $m$ is a perfect square  
(C) $m$ is a power of 3  
(D) $m$ is prime  
(E) $\tau(m)$ is 3
21. The CDC recently ran a program to immunize children against cooties. 1.6 million children were immunized over the 100-day program and their age, height, and weight (rounding to the nearest integer) were recorded. If the ranges of this data were:

<table>
<thead>
<tr>
<th></th>
<th>minimum</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (months)</td>
<td>13</td>
<td>142</td>
</tr>
<tr>
<td>Height (cm)</td>
<td>31</td>
<td>150</td>
</tr>
<tr>
<td>Weight (kg)</td>
<td>1</td>
<td>110</td>
</tr>
</tbody>
</table>

then how many of the following statements must be true? (In these statements, "age," "height," and "weight" refer to the child's recorded age, height, and weight.)

I. At least one child was immunized every day of the program.
II. One day during the program, two children with the same age, height, and weight were immunized.
III. One day during the program, two children with the same age and height were immunized.
IV. One day during the program, two children with the same height and weight were immunized.
V. One day during the program, two children with the same age and weight were immunized.
VI. Two children with the same age, height, and weight were immunized during the program.

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

22. For $x > 0$ one can write $\left(\frac{x^{7/2}}{x^{2/3}}\right)^{-6}$ as

(A) $x^{17}$  (B) 1  (C) $x^{25}$
(D) $\frac{1}{x^{17}}$  (E) none of these

23. If $\frac{1}{x} + y = 2$ and $\frac{1}{y} + x = 3$, what's $\frac{y}{x}$?

(A) $\frac{\sqrt{3}}{1 - \sqrt{3}}$  (B) $\frac{2}{3}$  (C) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$
(D) $\frac{-\sqrt{3}}{1 + \sqrt{3}}$  (E) none of these

24. If $x$ and $y$ are odd integers, which of the following must be an odd integer?

(A) $x + y$  (B) $xy$  (C) $x/y$
(D) $(xy + 1)^2$  (E) none of these
25. Each letter or digit on the front panel of a microwave oven is comprised of seven individual LED’s in the configuration shown. Each LED may be activated (on) or not (off) according to what letter or digit the configuration is to represent. When activated, an LED glows brightly and forms an individual segment of a letter or digit.

A power surge causes the individual LED’s to be activated in a random fashion with each segment having a probability of $\frac{5}{10^7}$ of being activated. Find the probability that the configuration of the LED’s that are activated causes an even digit to appear on the panel.

(A) $\frac{1}{2^5}$  
(B) $\frac{1}{10^7}$  
(C) $\frac{5}{10^7}$  
(D) $\frac{5}{2^7}$  
(E) none of these