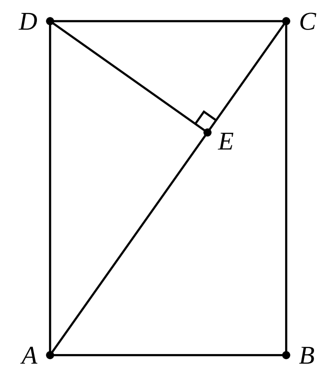
College of Charleston
Math Meet 2018
Written Test – Level 1

1. How many primes are one less than a perfect square?

   (A) none of them (B) 1 (C) 6 (D) 33 (E) infinitely many

2. Find the area of the rectangle $ABCD$ if (the length) $\overline{EC} = 8$, and $\overline{AE} = \overline{ED} \times \sqrt{2}$.
   (Figure not to scale.)



   (A) $192\sqrt{2}$ (B) $196\sqrt{2}$ (C) $198\sqrt{2}$ (D) $188\sqrt{2}$ (E) $164\sqrt{2}$

3. A teacher tried to divide a bag of pennies among her 3 favorite students, but after she gave each of them the same number of pennies, there was one penny left over. She then tried to divide the pennies equally among her 5 favorite students, but there was a penny left over, then among her 7 favorites, but there was a penny left over. Assuming she had more than one penny, what's the fewest number of pennies she could have had in her bag? The first (leftmost) digit in the correct answer is

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. Claire is a little dizzy from too much sun at the beach and she starts walking in a strange way:

   - she takes one step forward,
   - she turns 90° to her right and then takes two steps forward,
   - she turns 90° to her right and then takes one step forward,
   - she turns 90° to her left and then takes one step backward,
   - she starts all over again.

   Each step is 1 yard. After 186 steps, Claire passes out. How many yards from where she started does Claire end up?

   (A) 186 (B) 1 (C) 2 (D) $\sqrt{2}$ (E) $\sqrt{5}$

5. Farmer Mary keeps four kinds of animals: cows, chickens, ducks, and pigs. Next year, she would like to have 14 cows and chickens (that is, her cows and chickens should total 14), 12 chickens and ducks, 8 ducks and pigs, and 10 pigs and cows. What's the total number of animals must she have altogether?

   (A) 22

   (B) 32

   (C) 42

   (D) It's impossible for her to do this.

   (E) It's possible for her to do this, but there's more than one possible total.

6. When the polynomial $p(x)$ is divided by $x^2 - 1$, the remainder is $x + 2$. When $p(x)$ is divided by $x^2 - 4$, the remainder is $x + 1$. Find the remainder when $p(x)$ is divided by $(x - 1)(x - 2)$.

(A) $(x + 1)(x + 2)$ (B) $x + 1$ (C) $x - 1$

(D) $x + 2$ (E) 3

7. In a tennis tournament, n women and $2n$ men play, and each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number of matches won by men is 7 to 5, then n equals

(A) 3 (B) 8 (C) 11

(D) 16 (E) none of these

8. Two circles are concentric. A chord x units long cuts across the larger circle and is tangent to the smaller circle. Express the area of the region outside of the smaller circle , but inside the larger circle, in terms of x

(A) $x\pi/2$ (B) $2x\pi$ (C) $2x^2\pi$

(D) $x^2\pi/4$ (E) none of these

9. Two opposite edges of a tetrahedron are perpendicular, have lengths 7 units and 8 units respectively, and the distance between them (as measured along a line segment perpendicular to both edges) is 6 units. In cubic units, what is the volume of the tetrahedron?

(A) 56 (B) 108

(C) 168 (D) 336

(E) not enough information to tell

10. Which of the following is a solution to $x^6 - 5x^4 - 5x^2 + 1 = 0$?

(A) 6 (B) $\sqrt{5}$

(C) $3 + \sqrt{5}$ (D) $1 + \sqrt{2}$

(E) none of these is a solution

11. The 2003 inhabitants of an island are divided in two groups: the "truth tellers", who always tell the truth, and the "liars", who always lie. Each person is exactly one of the following: a singer, a soccer player or a fisherman. We ask each inhabitant the following three questions: 1) Are you a singer? 2) Are you a soccer player? 3) Are you a fisherman? 1000 people answer "yes" to the first question, 700 people answer "yes" to the second question, 500 people answer "yes" to the third question. How many "liars" are there on the island?

(A) 105 (B) 183 (C) 197 (D) 319 (E) 732

12. Let x_1, x_2, x_3 and x_4 be the four solutions to the equation $x^4 - 29x^2 + 100 = 0$, ordered so that $x_1 < x_2 < x_3 < x_4$. What is $x_2 + x_4$?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

13. The Yankees and the Red Sox are having a playoff series to determine the American League champion. The series continues until one team has won three games, and so may take up to five games. Assuming each team has an equal chance to win each game, what is the expected number of games in the series?

(A) $\dfrac{17}{4}$ (B) $\dfrac{15}{4}$ (C) $\dfrac{25}{8}$ (D) $\dfrac{33}{8}$ (E) 4

14. If Paul has fish for dinner, then he'll either pay his rent or read a book (or both). If he pays his rent, then he'll sleep in on Monday. If he does not read a book, then he'll not sleep in on Monday. Paul does not sleep in on Monday. What can you conclude?

(A) Paul paid his rent.

(B) If he does not have fish for dinner, then he'll read a book.

(C) If he pays his rent, then he'll *not* have fish for dinner.

(D) If he reads a book, then he'll sleep late on Monday.

(E) The sentence in answer d. is false.

15. How many pairs of positive integers (m, n) satisfy $\frac{1}{m} - \frac{1}{n} = \frac{1}{12}$?

(A) There are no such pairs (B) 3

(C) 7 (D) 12

(E) There are infinitely many such pairs

16. A 100 pound watermelon is 95 percent water. It is dehydrated until it is 80 percent water. What is its weight after dehydration?

(A) 20 pounds (B) 25 pounds (C) 50 pounds

(D) 80 pounds (E) none of these

17. If an equilateral triangle and a regular hexagon have the same perimeter, what is the ratio of the area of the triangle to the area of the hexagon?

(A) 1 to 6 (B) 1 to 3 (C) 2 to 3

(D) 5 to 6 (E) none of these

18. A math teacher has two identical pieces of milk chocolate and two identical pieces of dark chocolate to distribute to three students. Not every student must receive a piece of candy, but every piece must be given to student. How many different distributions are possible?

(A) 18 (B) 27 (C) 36 (D) 48 (E) 81

19. There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on the surface of each ball which is exactly 10 inches from each wall which that ball touches and 10 inches from the floor, then what is the sum of the radii of the balls?

(A) 15 inches (B) 20 inches (C) 25 inches

(D) 30 inches (E) not enough information

20. Figure 1 shows a map of eight cities, A,B,C,D,E,F,G,H, and routes for ten possible roads between the cities. A contractor wants to build as few of these roads as necessary so that it will be possible to go from any of the eight cities to any other using the finished roads. See, for example, Figure 2. How many ways are there to do this?

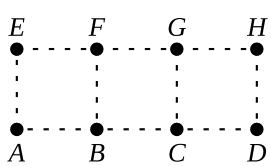


Fig. 1

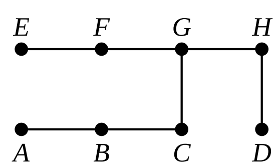


Fig. 2

(A) 120 (B) 69 (C) 56 (D) 48 (E) 35

21. If A and B are subsets of the set U, then

- $A \cap B$ is the set of all elements that belong to both A **and** B,
- $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
- A' is the set of all elements of U that are **not** in A.

If A, B, and C are subsets of U. Which of these equals

$$(A' \cup B) \cap (B' \cup C) \cap (A \cup C)?$$

(A) $(A \cup C)' \cup (B \cap C)$ (B) $(A \cap C') \cup (B \cup C)'$ (C) $(A \cup C)' \cup (B \cap C')$

(D) $(A \cap C) \cup (B \cup C')'$ (E) $(A \cup C')' \cup (B \cap C)$

22. Simplify $(\sqrt{2}-1)^1 + (\sqrt{2}-1)^2 + (\sqrt{2}-1)^3 + \cdots + (\sqrt{2}-1)^{2015}$.

(A) $\dfrac{1-(\sqrt{2}-1)^{2015}}{\sqrt{2}}$ (B) $(\sqrt{2}-1)^{2016}$ (C) $\sqrt{2}(\sqrt{2}-1)^{2015}$

(D) 2^{1008} (E) none of these

23. Find the ternary (base 3) expansion of $\dfrac{1}{4}$.

(A) $0.02002002\ldots$ (B) $0.020102010201\ldots$ (C) $0.02020202\ldots$

(D) $0.0201010101\ldots$ (E) $0.0200100200100\ldots$

24. The total paid for lunch by a group of 4 people was 60 dollars. The first person paid half of the sum of the amounts paid by the others. The second person paid one-third of the sum of the amounts paid by the others. The third person paid one-fourth of the sum of the amounts paid by the others. The second person paid how much more than the fourth person?

(A) 1 dollar (B) 2 dollars (C) 3 dollars

(D) 4 dollars (E) none of these

25. How many distinct real numbers x have the property that the median of the seven numbers $1.4, 1.6, 2.1, 2.2, 2.3, 2.4, x$ is equal to their mean?

(A) 0 (B) 1

(C) 2 (D) 3

(E) The are infinitely many such x.

2018 Answers / Level 1 Test

1. B
2. A
3. A
4. C
5. A
6. E
7. A
8. D
9. A
10. D
11. C
12. A
13. D
14. C
15. C
16. B
17. C
18. C
19. D
20. C
21. E
22. A
23. C
24. B
25. B