1. A student takes a 20 question multiple choice exam and answers every question. Each of the questions has five possible answers. In how many ways can the student answer all 20 questions incorrectly?
   (A) 1            (B) 20            (C) $4^{20}$
   (D) $5^{20}$    (E) none of these

2. Two cars starting from the same point each travel for 3 miles on a straight line going in opposite directions before they turn left and travel 4 more miles each and then stop. What is the distance between them when they stop?
   (A) 6 miles      (B) 8 miles      (C) 10 miles
   (D) 14 miles     (E) 24 miles

3. An ordinary six-sided die is thrown until a 5 appears. If a 5 does not appear on the first throw, find the probability that more than 3 throws will be needed.
   (A) $\frac{1}{36}$   (B) $\frac{25}{36}$   (C) $\frac{1}{216}$   (D) $\frac{125}{126}$   (E) $\frac{1}{126}$

4. Which of these is equal to
   \[
   \left( \frac{x^2 + 1}{x} \right) \left( \frac{y^2 + 1}{y} \right) + \left( \frac{x^2 - 1}{y} \right) \left( \frac{y^2 - 1}{x} \right)
   \]

   when $xy \neq 0$?
   (A) 1            (B) $2xy$         (C) $2x^2y^2 + 2$
   (D) $2xy + \frac{2}{xy}$   (E) $\frac{2x}{y} + \frac{2y}{x}$

5. Find the sum of the squares of the $x$ coordinates of all points of intersection of the graphs
   \[ f(x) = x \quad \text{and} \quad g(x) = 5x - x^3. \]
   (A) 0            (B) 1            (C) 4            (D) 5            (E) 8

6. How many subsets does $S = A \cup B$ have if $A = \{0, 1, -1, 2\}$ and $B = \{1, 2, 5\}$?
   (A) 8            (B) 13           (C) 32           (D) 64           (E) 7

7. Evaluate
   \[ 100^2 - 99^2 + 98^2 - 97^2 + \cdots - 3^2 + 2^2 - 1^2. \]
   (A) 2875         (B) 3038         (C) 4950         (D) 5050         (E) none of the above
8. Let \([x]\) denote the greatest integer \(n\) such that \(n \leq x\) and define the function
\[
f(x) = ([x] + [-x])^5.
\]
Which of these best describes the range of the function \(f\)?
(A) The range of \(f\) is the set of all real numbers.
(B) The range of \(f\) is an open interval.
(C) The range of \(f\) is a closed interval.
(D) The range of \(f\) is a set containing exactly one element.
(E) The range of \(f\) is a set containing exactly two elements.

9. An open rectangular box with square base and no top must have a volume of 100 cubic feet. The sides cost \(\$4\) per square foot and the bottom costs \(\$6\) per square foot to construct. Let \(x\) be the length of one of the edges of the bottom. Find an expression for the cost of constructing the box as a function of \(x\).
(A) \(4x^2 + \frac{24}{x}\)  
(B) \(6x^2 + 4x\)  
(C) \(22x\)  
(D) \(22x^2\)  
(E) \(6x^2 + \frac{1600}{x}\)

10. Which of these angles (measured in radians) has a tangent greater than zero and a cosine less than zero?
(A) \(\pi\)  
(B) \(-\pi/82\)  
(C) \(39\pi/4\)  
(D) \(27\pi/4\)  
(E) \(29\pi/4\)

11. Define the numbers \(a_0, \ldots, a_{2n}\) by the formula
\[
(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}.
\]
What is the value of the sum
\[
a_0 + a_2 + a_4 + \cdots + a_{2n}?
\]
(A) \(2^n\)  
(B) \(2^n + 1\)  
(C) \(\frac{3^n - 1}{2}\)  
(D) \(\frac{3^n}{2}\)  
(E) \(\frac{3^n + 1}{2}\)

12. If \(a = \sqrt{1 + \sqrt{1 + \sqrt{1}}}\) then \(a^4\) is
(A) \(\sqrt{2} + \sqrt{3}\)  
(B) \(\frac{1}{2}(7 + 3\sqrt{5})\)  
(C) \(1 + 2\sqrt{3}\)  
(D) \(3\)  
(E) \(3 + 2\sqrt{2}\)
13. What is the distance between the point \((x, y)\) on the line \(3x - y + 12 = 0\) and the point \((6, 10)\)?

(A) \(x + y - 16\)  
(B) \(\sqrt{10(x^2 + 4)}\)  
(C) \(\sqrt{(6 - x)^2 + (10 + y)^2}\)  
(D) \(\sqrt{(6 - x)^2 + 4}\)  
(E) \(\sqrt{10x^2 - 24x + 40}\)

14. Which of these is equal to \(\sqrt{3 + 2 \sqrt{2}} - \sqrt{3 - 2 \sqrt{2}}\)?

(A) 2  
(B) \(2\sqrt{3}\)  
(C) \(4\sqrt{2}\)  
(D) \(\sqrt{6}\)  
(E) 4

15. Suppose \(k\) is a fixed positive number in the interval \((0, 1)\). What is the sum of all numbers \(\theta\) in the interval \((0, 2\pi)\) such that \(\cos(\theta) = k\)?

(A) \(\frac{\pi}{2}\)  
(B) \(\pi\)  
(C) \(\frac{3\pi}{2}\)  
(D) \(2\pi\)  
(E) Can be different values depending on the value of \(k\).

16. The real-valued functions \(f, g\) and \(h\) are defined on the whole real line as follows:

\[f(x) = x + 1, \quad h(x) = 2x - 1, \quad g \circ f = h.\]

What is \(g(x)\)?

(A) \(g(x) = 2x\)  
(B) \(g(x) = \frac{1}{x}\)  
(C) \(g(x) = 2x - 3\)  
(D) \(g(x) = x^2 - 1\)  
(E) \(g(x) = 2x + 3\)

17. Given two numbers \(x\) and \(y\) with \(0 < x < 1\) and \(0 < y < 1\), which of the following statements is always true?

(A) \(x + y < 1\)  
(B) \(\frac{x}{y} < 1\)  
(C) \(x + y > 1\)  
(D) \(x^2 + y^2 > 1\)  
(E) \(x < \frac{1}{y}\)

18. How many two-digit numbers of the form \(AB\) (where \(A\) and \(B\) are the digits) satisfy \((AB)^2 = CAA\), where the digit \(C\) is \(B - 1\) (in decimal notation)?

(A) 0  
(B) 1  
(C) 2  
(D) 3  
(E) 9
19. How many values of \( \theta \) in \([0, \pi]\) satisfy both \( \sin 2\theta \cos \theta = \frac{2}{3} \) and \( \sin \theta \cos 2\theta = \frac{1}{3} \)?
(A) 0    (B) 1    (C) 2    (D) 3    (E) 4

20. Let \( x \) be a real number between .9 and 1. List the numbers \( x, y \) and \( z \) in order of increasing magnitude where

\[
y = x^x \quad \text{and} \quad z = x^y.
\]

(A) \( x < y < z \)    (B) \( x < z < y \)    (C) \( z < x < y \)
(D) \( z < y < x \)    (E) \( y < x < z \)

21. In the figure, \( AB \) is a diameter of the circle with center \( O \) and radius \( r \). A chord \( AD \) is drawn and extended until it meets the tangent to the circle at \( B \) in point \( C \). Then, point \( E \) is taken on \( AC \) so that \( AE = DC \). Let \( x \) be the minimum distance from \( E \) to the tangent though \( A \) and \( y \) be the minimum distance from \( E \) to the diameter \( AB \). What must be true about \( x \) and \( y \)?

\[
(A) \quad y^2 = \frac{x^3}{2r - x} \quad \quad (B) \quad y^2 = \frac{x^3}{2r + x} \quad \quad (C) \quad y^4 = \frac{x^2}{2r - x}
\]
\[
(D) \quad x^2 = \frac{y^2}{2r - x} \quad \quad (E) \quad x^2 = \frac{y^2}{2r + x}
\]

22. Find the length of the radius of a circle in which a chord of length 6 is twice as far from the center as a chord of length 12.

(A) \( 3\sqrt{5} \)    (B) \( 5\sqrt{3} \)    (C) \( 2\sqrt{6} \)    (D) \( 6\sqrt{2} \)    (E) \( \sqrt{30} \)

23. Let \( x \) and \( y \) be nonzero real numbers for which

\[
x + \frac{6}{x} = 2y + \frac{3}{y}.
\]

If \( x/y \neq 2 \), find the product \( xy \).

(A) \(-3\)    (B) \(-1\)    (C) \(1\)    (D) \(3\)    (E) \(\sqrt{5}\)
24. The diagram shows the dimensions of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that fits inside the room?

![Diagram of an L-shaped room with dimensions 3, 10, 12, and 18]

- (A) $16\pi$
- (B) $\frac{81}{4}\pi$
- (C) $25\pi$
- (D) $\frac{145}{4}\pi$
- (E) $841\pi$

25. An interior point of an equilateral triangle is at distances 5, 7, and 8 from the three sides of the triangle. What is the common length of the sides of the triangle?

- (A) It cannot be determined
- (B) The given configuration cannot exist
- (C) 20
- (D) $14\sqrt{3}$
- (E) $\frac{40}{3}\sqrt{3}$
Answers

1. c
2. c
3. b
4. d
5. e
6. c
7. d
8. e
9. e
10. e
11. e
12. e
13. b
14. a
15. d
16. c
17. e
18. b
19. a
20. b
21. a
22. a
23. d
24. c
25. e