# College of Charleston
# Math Meet 2004
# Written Test – Level 2

1. A student takes a 20 question multiple choice exam and answers every question. Each of the questions has five possible answers. In how many ways can the student answer all 20 questions incorrectly?

   (A) $1$ (B) $20$ (C) $4^{20}$ (D) $5^{20}$ (E) none of these

2. Two cars starting from the same point each travel for 3 miles on a straight line going in opposite directions before they turn left and travel 4 more miles each and then stop. What is the distance between them when they stop?

   (A) 6 miles (B) 8 miles (C) 10 miles (D) 14 miles (E) 24 miles

3. An ordinary six-sided die is thrown until a $5$ appears. If a $5$ does not appear on the first throw, find the probability that more than $3$ throws will be needed.

   (A) $\frac{1}{36}$ (B) $\frac{25}{36}$ (C) $\frac{1}{216}$ (D) $\frac{125}{126}$ (E) $\frac{1}{126}$

4. Which of these is equal to

$$\left(\frac{x^2+1}{x}\right)\left(\frac{y^2+1}{y}\right)+\left(\frac{x^2-1}{y}\right)\left(\frac{y^2-1}{x}\right)$$

   when $xy \neq 0$?

   (A) $1$ (B) $2xy$ (C) $2x^2y^2+2$ (D) $2xy+\frac{2}{xy}$ (E) $\frac{2x}{y}+\frac{2y}{x}$

5. Find the sum of the squares of the $x$ coordinates of all points of intersection of the graphs

$$f(x)=x \quad \text{and} \quad g(x)=5x-x^3.$$

   (A) $0$ (B) $1$ (C) $4$ (D) $5$ (E) $8$

6. How many subsets does $S = A \cup B$ have if $A = \{0,1,-1,2\}$ and $B=\{1,2,5\}$?

   (A) $8$ (B) $13$ (C) $32$ (D) $64$ (E) $7$

7. Evaluate

$$100^2-99^2+98^2-97^2+\cdots-3^2+2^2-1^2.$$

   (A) $2875$ (B) $3038$ (C) $4950$ (D) $5050$ (E) none of the above

8. Let $[x]$ denote the greatest integer $n$ such that $n \leq x$ and define the function

$$f(x) = \left([x] + [-x]\right)^5.$$

Which of these best describes the *range* of the function $f$?

(A) The range of $f$ is the set of all real numbers.

(B) The range of $f$ is an open interval.

(C) The range of $f$ is a closed interval.

(D) The range of $f$ is a set containing exactly one element.

(E) The range of $f$ is a set containing exactly two elements.

9. An open rectangular box with square base and no top must have a volume of 100 cubic feet. The sides cost $\$4$ per square foot and the bottom costs $\$6$ per square foot to construct. Let $x$ be the length of one of the edges of the bottom. Find an expression for the cost of constructing the box as a function of $x$.

(A) $4x^2 + \dfrac{24}{x}$ (B) $6x^2 + 4x$ (C) $22x$

(D) $22x^2$ (E) $6x^2 + \dfrac{1600}{x}$

10. Which of these angles (measured in radians) has a tangent greater than zero and a cosine less than zero?

(A) $\pi$ (B) $-\pi/82$ (C) $39\pi/4$

(D) $27\pi/4$ (E) $29\pi/4$

11. Define the numbers $a_0, \ldots, a_{2n}$ by the formula

$$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}.$$

What is the value of the sum

$$a_0 + a_2 + a_4 + \cdots + a_{2n}?$$

(A) $2^n$ (B) $2^n + 1$ (C) $\dfrac{3^n - 1}{2}$

(D) $\dfrac{3^n}{2}$ (E) $\dfrac{3^n + 1}{2}$

12. If $a = \sqrt{1 + \sqrt{1 + \sqrt{1}}}$ then $a^4$ is

(A) $\sqrt{2} + \sqrt{3}$ (B) $\dfrac{1}{2}(7 + 3\sqrt{5})$ (C) $1 + 2\sqrt{3}$

(D) $3$ (E) $3 + 2\sqrt{2}$

13. What is the distance between the point $(x, y)$ on the line $3x - y + 12 = 0$ and the point $(6, 10)$?

(A) $x + y - 16$ (B) $\sqrt{10(x^2 + 4)}$

(C) $\sqrt{(6 - x)^2 + (10 + y)^2}$ (D) $\sqrt{(6 - x)^2 + 4}$

(E) $\sqrt{10x^2 - 24x + 40}$

14. Which of these is equal to $\sqrt{3 + 2\sqrt{2}} - \sqrt{3 - 2\sqrt{2}}$?

(A) $2$ (B) $2\sqrt{3}$ (C) $4\sqrt{2}$ (D) $\sqrt{6}$ (E) $4$

15. Suppose $k$ is a fixed positive number in the interval $(0, 1)$. What is the sum of all numbers $\theta$ in the interval $(0, 2\pi)$ such that $\cos(\theta) = k$?

(A) $\dfrac{\pi}{2}$

(B) $\pi$

(C) $\dfrac{3\pi}{2}$

(D) $2\pi$

(E) Can be different values depending on the value of $k$.

16. The real-valued functions $f$, $g$ and $h$ are defined on the whole real line as follows:

$$f(x) = x + 1, \qquad h(x) = 2x - 1, \qquad g \circ f = h.$$

What is $g(x)$?

(A) $g(x) = 2x$ (B) $g(x) = \dfrac{1}{x}$

(C) $g(x) = 2x - 3$ (D) $g(x) = x^2 - 1$

(E) $g(x) = 2x + 3$

17. Given two numbers $x$ and $y$ with $0 < x < 1$ and $0 < y < 1$, which of the following statements is always true?

(A) $x + y < 1$ (B) $\dfrac{x}{y} < 1$ (C) $x + y > 1$

(D) $x^2 + y^2 > 1$ (E) $x < \dfrac{1}{y}$

18. How many two-digit numbers of the form $AB$ (where $A$ and $B$ are the digits) satify $(AB)^2 = CAAB$, where the digit $C$ is $B - 1$ (in decimal notation)?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 9

19. How many values of $\theta$ in $[0, \pi]$ satisfy both $\sin 2\theta \cos \theta = 2/3$ and $\sin \theta \cos 2\theta = 1/3$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

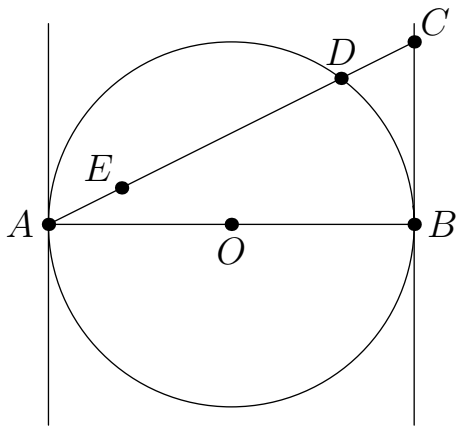
20. Let $x$ be a real number between $.9$ and $1$. List the numbers $x$, $y$ and $z$ in order of increasing magnitude where

$$y = x^x \quad \text{and} \quad z = x^y.$$

(A) $x < y < z$ (B) $x < z < y$ (C) $z < x < y$

(D) $z < y < x$ (E) $y < x < z$

21. In the figure, $AB$ is a diameter of the circle with center $O$ and radius $r$. A chord $AD$ is drawn and extended until it meets the tangent to the circle at $B$ in point $C$. Then, point $E$ is taken on $AC$ so that $AE = DC$. Let $x$ be the minimum distance from $E$ to the tangent though $A$ and $y$ be the minimum distance from $E$ to the diameter $AB$. What must be true about $x$ and $y$?




(A) $y^2 = \dfrac{x^3}{2r - x}$ (B) $y^2 = \dfrac{x^3}{2r + x}$ (C) $y^4 = \dfrac{x^2}{2r - x}$

(D) $x^2 = \dfrac{y^2}{2r - x}$ (E) $x^2 = \dfrac{y^2}{2r + x}$

22. Find the length of the radius of a circle in which a chord of length $6$ is twice as far from the center as a chord of length $12$.

(A) $3\sqrt{5}$ (B) $5\sqrt{3}$ (C) $2\sqrt{6}$ (D) $6\sqrt{2}$ (E) $\sqrt{30}$

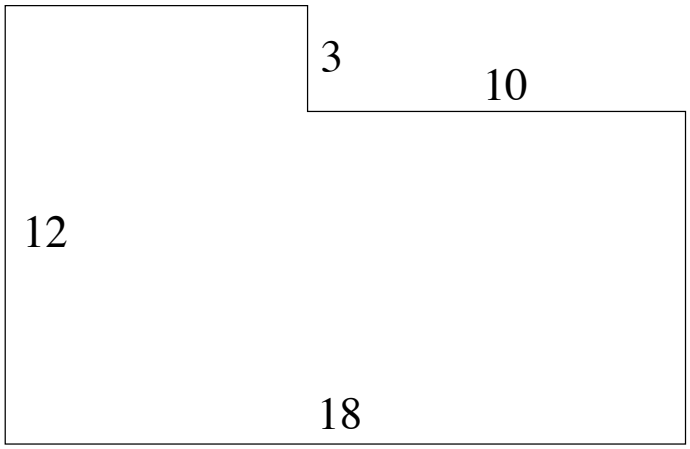
23. Let $x$ and $y$ be nonzero real numbers for which

$$x + \frac{6}{x} = 2y + \frac{3}{y}.$$

If $x/y \neq 2$, find the product $xy$.

(A) $-3$ (B) $-1$ (C) $1$ (D) $3$ (E) $\sqrt{5}$

24. The diagram shows the dimensions of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that fits inside the room?




(A) $16\pi$ (B) $\frac{81}{4}\pi$ (C) $25\pi$ (D) $\frac{145}{4}\pi$ (E) $841\pi$

25. An interior point of an equilateral triangle is at distances $5$, $7$ and $8$ from the three sides of the triangle. What is the common length of the sides of the triangle?

(A) It cannot be determined

(B) The given configuration cannot exist

(C) $20$

(D) $14\sqrt{3}$

(E) $\frac{40}{3}\sqrt{3}$

# Answers

1. c
2. c
3. b
4. d
5. e
6. c
7. d
8. e
9. e
10. e
11. e
12. e
13. b
14. a
15. d
16. c
17. e
18. b
19. a
20. b
21. a
22. a
23. d
24. c
25. e