

College of Charleston
Math Meet 2006
Written Test – Level 2

1. If $f(x) = \ln(x^2)$ and $g(x) = 2 \ln(x)$, then which of the following is true:
(A) $f(x) = g(x)$ for all real numbers x .
(B) $f(x) = g(x)$ for all real numbers in the domain of g .
(C) $f(x) = g(x)$ for all real numbers in the domain of f .
(D) $f(x) = g(x)$ except for $x = 0$.
(E) $f(x) \neq g(x)$ for all real numbers x .
2. ToLoose LaTrack is a realtor who rents office space in a building. The rent is a linear function of the area of the office. The rent for a 600-square-foot office is 950 dollars per month while the rent for a 900-square-foot office is 1350 dollars per month. Find the monthly rent that ToLoose would charge for a 1200-square-foot office.
(A) 900 dollars per month (B) 1450 dollars per month
(C) 1750 dollars per month (D) 1600 dollars per month
(E) None of the above.
3. What is the value of $\arctan(\cot(2\pi/3))$?
(A) $\pi/6$ (B) $-\pi/6$
(C) $2\pi/3$ (D) $-\pi/3$
(E) None of the above
4. A commuter train currently carries 600 passengers from a suburb to a city. It now costs 1 dollar per passenger to ride the train. A study shows that 50 additional people will ride the train for each 5 cent reduction in fare. What fare should be charged to maximize revenue?
(A) 95 cents (B) 90 cents (C) 85 cents
(D) 80 cents (E) 75 cents
5. A box contains five red balls and seven blue balls. A ball is selected at random and discarded. A second ball is then selected at random from the remaining balls. The probability that this second selection is a blue ball is
(A) $7/11$ (B) $5/11$ (C) $7/12$
(D) $5/12$ (E) none of these

6. If $p(x)$ is a polynomial, and if $p(x) + ip(1-x) = \sqrt{2}x^5$, what's the coefficient of x^5 when $p(x)$ is written in the standard power form $p(x) = a + bx + cx^2 + \dots$? (Here, i denotes $\sqrt{-1}$.)

- (A) $1 - i$ (B) $\sqrt{2}(1 - i)$ (C) $\sqrt{2}(1 + i)$
 (D) $\frac{1 - i}{\sqrt{2}}$ (E) $\frac{1 + i}{\sqrt{2}}$

7. What is $\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{41}$?

- (A) $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$
 (C) $-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ (D) $-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$
 (E) None of the above

8. Two adjacent sides of a triangle have lengths 3 and 7 respectively, and the interior angle between them is 60 degrees. Find the length of the third side. Round your answer to the nearest whole number.

- (A) 5 (B) 6
 (C) 7 (D) 8
 (E) None of the above

9. In a polytropic atmosphere, temperature T and pressure p are related by the following equation, where T_0 is the temperature at pressure p_0 :

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}.$$

Solve the above relation for k

- (A) $k = \frac{-\log p + \log p_0}{\log p + \log p_0 + \log T - \log T_0}$
 (B) $k = \frac{\log p - \log p_0 - \log T + \log T_0}{\log p - \log p_0}$
 (C) $k = \frac{\log p_0 - \log p + \log T - \log T_0}{\log p + \log p_0}$
 (D) $k = \frac{\log p - \log p_0}{\log p - \log p_0 - \log T + \log T_0}$
 (E) None of the above

10. Perform the multiplication

$$e^{-i\pi/4}(1+i).$$

- (A) $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$
(C) $\sqrt{2}$ (D) 1
(E) None of the above

11. Construct the cubic real polynomial with leading coefficient 1 and with two of its roots being 3 and $1+i$. What is the coefficient of x^2 ?

- (A) -5 (B) 5
(C) -3 (D) 3
(E) None of the above

12. A toy rocket is launched by a blast of compressed air. The height in feet of the rocket t seconds after lift off is given by

$$h = vt - 16t^2,$$

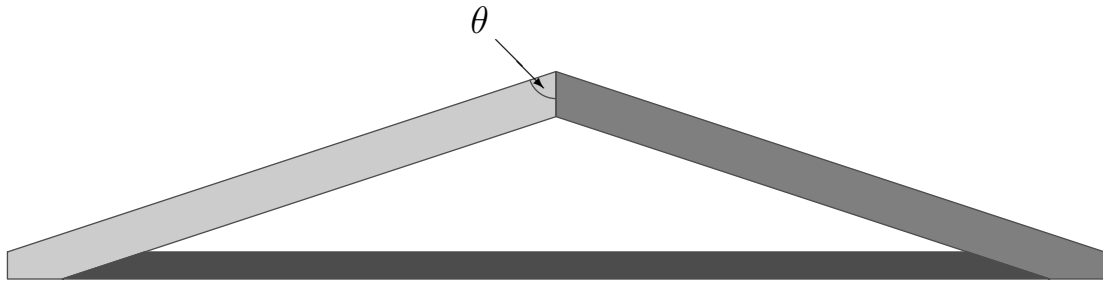
where v is the initial velocity. If the time between lift off and the rocket returning to the ground is T seconds, what was the maximum height that the rocket reached?

- (A) $32T$ (B) $4T^2$
(C) $8T^2$ (D) $8T$
(E) none of the above

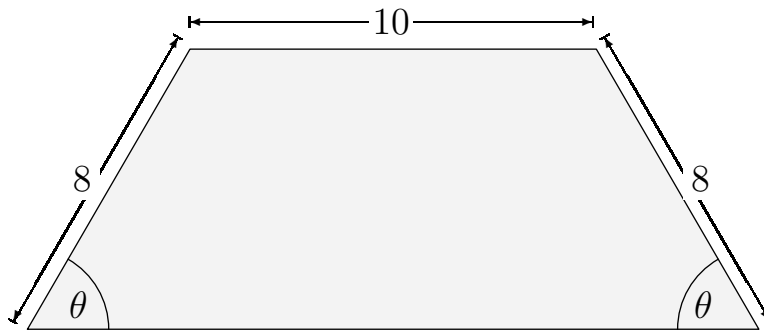
13. A pharmacist working the night shift at a hospital receives a call from the operating room that they have an urgent need for 2 liters of 74% alcohol solution. She looks in the alcohol vault and finds that they only have 90% and 70% solutions in stock, which she will have to mix together to get the 74% solution. How much of the 70% solution should she use?

- (A) 1.3 liters (B) 1.4 liters (C) 1.5 liters
(D) 1.6 liters (E) 1.7 liters

14. Three boards are to be cut and nailed together as shown in the diagram to form the rafter for a roof. The roof should have a slope of $1/3$. What is the angle θ at which the top left board should be cut?



- (A) $\tan^{-1} 1$ (B) $\sin^{-1} 3$ (C) $\cot^{-1} \frac{1}{3}$
 (D) $\sec^{-1} 3$ (E) $\tan^{-1} \frac{1}{3}$
15. Find the nearest integer to the solution of the equation $x + e^{2-1/\ln|x|} = 0$.
 (A) -3 (B) -1 (C) 0 (D) 1 (E) 3
16. A rectangular aquarium tank has width x feet, height twice its width, and length y feet. The glass for the sides and ends costs \$3 per square foot but the material for the base only costs \$2 per square foot. There is no top. Find an expression for the total cost of the materials to make the tank.
 (A) $14xy + 12x^2$ (B) $5xy + 4x^2$ (C) $12x + 4y$
 (D) $34x + 10y$ (E) $8xy + 6x^2$
17. Express the area of the trapezoid in terms of the angle θ .



- (A) $4(20 + 2 \tan \theta)$ (B) $80 \cos \theta + 64 \cos \theta \sin \theta$
 (C) $4 \sin \theta(10 + 2 \cos \theta)$ (D) $4 \tan \theta(20 + 16 \cot \theta)$
 (E) $80 \sin \theta + 32 \sin 2\theta$

18. Your employer offers two options for health insurance plans. Under Plan A you pay \$100 per month for premiums. You pay all of your medical expenses until they reach \$5000 in one year. Beyond that you pay 20% of your expenses and the insurance pays the rest for the remainder of the year. Under Plan B you pay \$300 per month for premiums. You pay all of your medical expenses until they reach \$400 in one year. Beyond that you pay 20% and the insurance company pays the rest for the remainder of the year. If you have low medical expenses then the total annual cost (total premiums for the year plus the part of your medical expenses that you pay for the year) is less under plan A but if you have higher medical expenses the total annual cost of plan B is less. What is the break even point, i.e. the level of medical expenses in a year where the total annual cost of both plans is the same?
 (A) \$3000 (B) \$3400 (C) \$3500 (D) \$5300 (E) \$5800
19. The base of a pyramid is an equilateral triangle, and the apex of the pyramid is directly above the center of the base. The three other sides of the pyramid are right-angled triangles. If the distance from the apex to each of the corners of the base is 4, what is the volume of the pyramid, when rounded to the nearest integer?
 (A) 8 (B) 11 (C) 18 (D) 55 (E) 64
20. Triangle ABC is not a right triangle. Side AB has length 3, side BC has length 8, and side CA has length 9. A line drawn from vertex B perpendicular to side AC intersects side AC at point D. Find the length of the line segment AD.
 (A) $\frac{13}{9}$ (B) $\frac{15}{8}$ (C) $\frac{7}{3}$
 (D) $\frac{7}{12}$ (E) none of these
21. Two vertices of a triangle lie on the ellipse given by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The third vertex is at the focus $(3, 0)$, and the opposite side of the triangle passes through the other focus. What is the perimeter of the triangle?
 (A) 10 (B) 20 (C) 50
 (D) 100 (E) none of these
22. The light rays from a spotlight spread out to form a solid cone, with the light rays making an angle of up to 30 degrees with the centerline of the beam. If the spotlight is 10 feet above the floor, and the centerline of the beam hits the floor at a 60-degree angle of inclination, what is the area the spotlight covers on the floor?
 (A) 25π (B) 75π (C) $100\pi/\sqrt{3}$
 (D) $400\pi/9$ (E) $25\sqrt{6}\pi$

23. In the four-by-four grid shown, each of the 4 two-by-two squares in the corner must contain 4 distinct integers, greater than one, whose sum is 17. No two numbers that are in the same row or column can have a common factor greater than one. What is the product of the four numbers on the diagonal?

		5	
2			
	3		
			3

- (A) 30 (B) 60 (C) 90
 (D) 210 (E) none of these
24. The points $(1, 3)$ and $(7, 11)$ are vertices of a square at opposite ends of a diagonal of the square. Find the coordinates of the other vertex of the square that has $x > 1$.
- (A) $(6, \frac{11}{2})$ (B) $(\frac{20}{3}, 5)$ (C) $(12, 1)$
 (D) $(8, 4)$ (E) none of these
25. For every nonnegative integer x , let $S(x)$ be the sequence of finitely many (possible zero) integers defined as follows.
 $S(0) = \{\}$, and if $x > 0$, then $S(x) = \left\{ r, S\left(\frac{x-r}{2}\right) \right\}$, where r is the remainder when x is divided by 2. That is, when x is 0, the sequence $S(x)$ has no terms, and when x is positive, the first term in the sequence $S(x)$ is r , and the remaining terms are those of $S\left(\frac{1}{2}(x-r)\right)$.
 For instance, $S(0) = \{\}$, $S(1) = \{1\}$, and $S(2) = \{0, 1\}$.
 If $S(x) = \{1, 1, 0, 0, 0, 1, 1\}$, what is x ?
- (A) 25 (B) 51 (C) 75 (D) 99 (E) 121

2006 Answers / Level 2 Test

1. B
2. C
3. B
4. D
5. C
6. E
7. A
8. B
9. D
10. C
11. A
12. B
13. D
14. C
15. A
16. A
17. E
18. B
19. B
20. A
21. B
22. E
23. C
24. D
25. D