

College of Charleston
Math Meet 2013
Written Test – Level 2

1. A function f is given by the formula

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ f(0)x + f(0) & \text{if } 0 < x \leq 1 \\ f(1)(x - 1) + f(1) & \text{if } 1 < x \leq 2 \\ f(2)(x - 2) + f(2) & \text{if } 2 < x \leq 3 \\ 7 & \text{if } x > 3 \end{cases}$$

Find $f(2.5)$.

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 8
2. What is the amplitude of $3 \sin(x) + 4 \cos(x)$?
 (A) 1 (B) 4 (C) 5
 (D) 7 (E) none of these
3. How many sequences of consecutive positive integers have a sum of 30?
Note added after Math Meet 2013: Include sequences of length one.
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. Let S_1 be the set of points (x, y) in the plane whose coordinates satisfy the equation

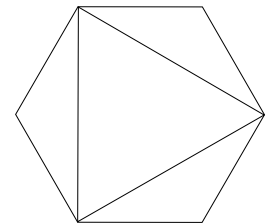
$$(x + y + 2)(x - y + 2) = 0$$

and let S_2 be the set of points satisfying

$$(x + y - 2)(y - 2) = 0.$$

How many points are there in the intersection $S_1 \cap S_2$ (the set of points satisfying both equations simultaneously)?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
5. A regular hexagon has side length 1. Using alternating vertices of the hexagon, we form an equilateral triangle. What is the area of the triangle?



- (A) $\frac{1}{2}$ (B) 6 (C) π
- (D) $\frac{3\sqrt{3}}{4}$ (E) none of these

6. Evaluate $\sum_{k=0}^{11} \cos \frac{k\pi}{6}$.

- (A) 6 (B) $\frac{11}{2}$ (C) $6\sqrt{3}$
 (D) $2\sqrt{3}$ (E) none of these

7. The graphs of the functions $f(x) = 2^x$ and $g(x) = 3^{-x}$ have
 (A) no points in common. (B) one common point on the x -axis.
 (C) the same y -intercept. (D) two points in common.
 (E) three points in common.

8. Which of these choices correctly lists these four numbers in ascending order?
 (A) $2^{500} < 3^{400} < 4^{300} < 5^{200}$ (B) $4^{300} < 3^{400} < 2^{500} < 5^{200}$
 (C) $5^{200} < 2^{500} < 4^{300} < 3^{400}$ (D) $3^{400} < 2^{500} < 4^{300} < 5^{200}$
 (E) $4^{300} < 5^{200} < 3^{400} < 2^{500}$

9. How is the area of a triangle changed if the length of the base of the triangle is *increased* by 10% and the height of the triangle is *decreased* by 10%?
 (A) Decreases by 5% (B) Decreases by 1% (C) Remains the same
 (D) Increases by 2% (E) Increases by 10%

10. If x is the number

$$x = (\log_8 2)^{(\log_2 8)}$$

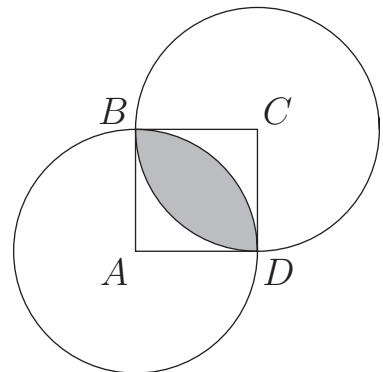
then what is $\log_3 x$?

- (A) -3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 3 (E) 9

11. A right triangle in the xy -plane has vertices at the points $(1, 4)$, $(5, -1)$ and $(1, -1)$. The triangle is rotated 90° clockwise around the origin, reflected across the x -axis, translated 4 units right and finally translated 3 units down. What are the coordinates of the midpoint of the hypotenuse of the triangle after these transformations?

- (A) $(3.5, -1)$ (B) $(0, 4.5)$ (C) $(-2, 2.5)$ (D) $(4.5, 1)$ (E) $(5.5, 0)$

12. The square $ABCD$ in the figure has sides of length 1. Both of the circles pass through the points B and D , but one is centered at A and the other at C . What is the area of the intersection of the two circles (the shaded region in the figure)?



- (A) $\frac{\pi}{4}$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{\pi}{2} - 1$ (D) $\frac{3\pi}{4} - 1$ (E) $\frac{1}{3}$

13. Find the quotient when $5x^4 - 2x^3 + 13x^2 + 7$ is divided by $x^2 + 2$.
 (A) $-4x + 1$ (B) $4x - 3$ (C) $5x^2 + 2x - 4$
 (D) $5x^2 - 2x + 3$ (E) None of these
14. At which of these times is the *product* of the numbers before and after the “:” equal to the number of degrees in the angle formed by the hour and minute hand on an analog clock?
 (A) 1:05 (B) 2:08 (C) 3:40 (D) 4:42 (E) 5:05
15. Let $f(1) = 2013$ and $f(n + 1) = f(n) + 2n - 1$. Evaluate $f(61)$.
 (A) 3355 (B) 4062 (C) 5613
 (D) 122793 (E) none of these
16. Simplify $(w + x + y - z)^2 + (x + y + z - w)^2 + (y + z + w - x)^2 + (z + w + x - y)^2$.
 (A) 0
 (B) $4(w^2 + x^2 + y^2 + z^2)$
 (C) $(x + y + z + w)^2$
 (D) $8(wx + wy + wz + xy + xz + yz)$
 (E) none of these
17. Given that the five digit number $738t8$ is divisible by 12 and sets $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{5, 6, 7, 8, 9\}$ and $C = \{3, 4, 5, 6, 7\}$, which set(s) contain(s) the possible value(s) of the digit t ?
 (A) Only A (B) Only B (C) A and B (D) B and C (E) C and A
18. Find all values of x in $[0, 2\pi]$ which satisfy the equation $\tan(2x) = -\sqrt{3}$.
 (A) $\pi/3, 5\pi/6$ (B) $4\pi/3, 11\pi/6$
 (C) $5\pi/12, 11\pi/12$ (D) $\pi/3, 5\pi/6, 4\pi/3, 11\pi/6$
 (E) $5\pi/12, 11\pi/12, 17\pi/12, 23\pi/12$
19. What is the probability of getting a sum of sixteen in tossing three standard dice?
 (A) $1/6$ (B) $1/27$ (C) $1/36$
 (D) $1/12$ (E) None of the above
20. Find the largest two-digit integer that is increased by 75% when its digits are reversed.
 (A) 24 (B) 27 (C) 36
 (D) 54 (E) None of the above.
21. Given the following four statements:
 I. Exactly one of these statements is false.
 II. Exactly two of these statements are false.
 III. Exactly three of these statements are false.
 IV. Exactly four of these statements are false
- Assume that each statement is either true or false. Among them, the number of false statements is exactly
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. Find the period of the function $\sin(2x) \sin(6x) \sin(8x)$.
 (A) $\pi/3$ (B) $\pi/4$ (C) $\pi/2$ (D) π (E) 2π
23. Which of the following is a factor of $x^{30} + y^{30}$:
 $x^2 + y^2$, $x^3 + y^3$ or $x^5 + y^5$?
 (A) only $x^2 + y^2$ (B) only $x^3 + y^3$
 (C) only $x^5 + y^5$ (D) all three are factors
 (E) none of the three is a factor
24. The function $\frac{x+1}{x^3+1}$ can be written as the sum of an even function and an odd function. Find the even function.
 (A) $\frac{x^4-1}{x^3+1}$ (B) $\frac{x^4+1}{x^6+1}$ (C) $\frac{x^4-1}{x^6+1}$ (D) $\frac{x^4+1}{x^6-1}$ (E) $\frac{x^4-1}{x^6-1}$
25. You have a hat with a white ball in it. You also have a paper bag with one white and one black ball in it. First, you randomly select one of the balls out of the bag (without looking at it) and place it in the hat. Then you randomly select one of the two balls out of the hat. If that ball is white, what is the probability that the ball still in the hat is also white?
 (A) 0 (B) $1/3$ (C) $1/2$ (D) $2/3$ (E) 1

2013 Answers / Level 2 Test

- | | | |
|------|-------|-------|
| 1. D | 10. A | 19. C |
| 2. C | 11. E | 20. E |
| 3. E | 12. C | 21. D |
| 4. C | 13. D | 22. C |
| 5. D | 14. B | 23. A |
| 6. E | 15. C | 24. E |
| 7. C | 16. B | 25. D |
| 8. C | 17. E | |
| 9. B | 18. D | |