College of Charleston
Math Meet 2019
Written Test – Level 2

1. Find the solution set: $|2x+1| + |x-4| \leq |x-6|$

   (A) $\left[-\frac{3}{2}, \frac{9}{4}\right]$ (B) $\left[-\frac{3}{2}, \frac{1}{2}\right]$ (C) $\left[\frac{1}{2}, \frac{9}{4}\right]$ (D) $\left[-\frac{1}{2}, \frac{9}{4}\right]$ (E) $\left[-\frac{9}{4}, \frac{3}{2}\right]$

2. On the interval $(-1, 1)$, the function $\dfrac{x-1}{x^3+1}$ can be written as a sum $f(x) + g(x)$ where f is even and g is odd. Find $f(x) - g(x)$.

   (A) $\dfrac{1}{x^2-x+1}$ (B) $\dfrac{1}{x^2+x+1}$ (C) $\dfrac{-1}{x^2+x+1}$

   (D) $\dfrac{1+x}{x^3-1}$ (E) $\dfrac{1-x}{x^3+1}$

3. Oman and Bar are playing a game. Each rolls a fair 6-sided die. (This means that each player receives a number in $\{1, 2, 3, 4, 5, 6\}$ each with equal probability.) If Oman rolls a 6 and Bar rolls a number less than 5, then Oman wins. If Oman rolls a number less than 6 and Bar rolls a 5 or 6, then Bar wins. In case of any other combination of rolls, the players reroll until someone wins. Find the probability that Oman will win.

   (A) 1/6 (B) 2/7 (C) 3/11 (D) 4/36 (E) 5/49

4. A telephone company offers two plans for long distance service. Plan A cost $\$15$ a month plus 7 cents per minute of long distance calls. Plan B has no monthly fee but costs 10 cents per minute of long distance calls. When is plan A less expensive than plan B?

   (A) If you talk for more than 8 hours and 20 minutes.

   (B) If you talk for more than 6 hours and 28 minutes.

   (C) If you talk for more than 2 hours and 30 minutes.

   (D) If you talk for less than 6 hours and 28 minutes.

   (E) If you talk for less than 2 hours and 30 minutes.

5. Which of these is always equal to $2\sin(x)\cos(x)$?

   (A) $\cos(2x)$

   (B) $\sin(x+\frac{\pi}{5})\cos(x-\frac{\pi}{5}) - \sin(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (C) $\sin(x+\frac{\pi}{5})\cos(x-\frac{\pi}{5}) + \sin(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (D) $\sin(x+\frac{\pi}{5})\sin(x-\frac{\pi}{5}) - \cos(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (E) $\sin(x+\frac{\pi}{5})\sin(x-\frac{\pi}{5}) + \cos(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

6. A sequence of numbers $a_0, a_1, a_2, \ldots$ satisfies the **recurrence relation**

$$a_{n+2} = a_n \text{ for all } n \geq 0.$$

Another sequence $b_0, b_1, b_2, \ldots$ satisfies

$$b_{n+1} = 2b_n \text{ for all } n \geq 0.$$

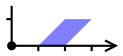
Let $c_n = a_n + b_n$ for all $n \geq 0$. Which of these recurrence relations must be true (for all $n \geq 0$)?

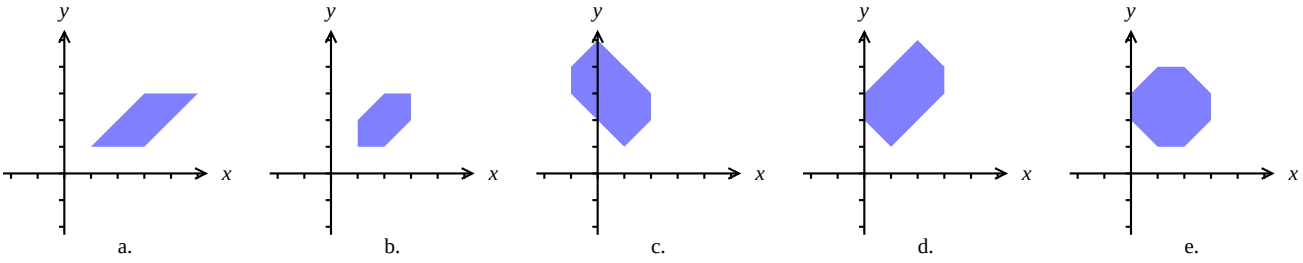
(A) $c_{n+2} = 2c_{n+1} + c_n$ (B) $c_{n+2} = c_{n+1} - 2c_n$

(C) $c_{n+2} = 5c_n$ (D) $c_{n+3} = 2c_{n+2} + c_{n+1} - 2c_n$

(E) $c_{n+3} = 2c_{n+2} - c_{n+1} + 2c_n$

7. When Rohn tries to throw a football, his aim is poor but very predictable. See the top figure below. If he stands on the dot ● and throws in the direction of the arrow, the ball will always land in the shaded parallelogram.
Suppose Rohn stands at the origin and, aiming in the positive horizontal direction, throws a ball. Then he walks to where the ball landed, aims in the positive vertical direction, and throws the ball again. Which figure shows the shaded region in which the ball could land the second time?





(A) Figure a. (B) Figure b. (C) Figure c. (D) Figure d. (E) Figure e.

8. Which of these polynomials is **not** a factor of $x^{10} - 1$?

(A) $x^5 - 1$ (B) $x^9 + x^8 + x^7 + \cdots + x + 1$

(C) $x^9 - x^8 + x^7 - \cdots + x - 1$ (D) $x^8 + x^6 + x^4 + x^2 + 1$

(E) $x^8 - x^6 + x^4 - x^2 + 1$

9. In a sales effectiveness seminar, representatives tried two different approaches to selling an HDTV to a customer, the aggressive approach and the passive approach. The table below contains the results for 1160 customers. Supose that one of these customers is selected at random. Let S represent the event that the customer purchased an HDTV and let A represent the event that the sales approach was aggressive.

	Sale	No Sale	Total
Aggressive	270	310	580
Passive	416	164	580
Total	686	474	1160

Which of these statements correctly characterizes the relationship between events S and A?

(A) they are disjoint and independent

(B) they are independent but not disjoint

(C) they are disjoint but not independent

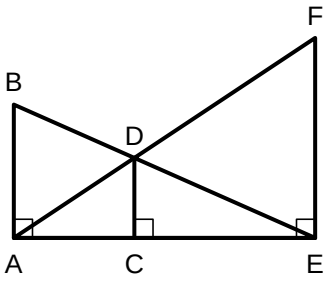
(D) they are neither disjoint nor independent

(E) they have the same probability

10. Which of these is a simpler way to write the number $\dfrac{\frac{1}{c-1}+1}{\frac{1}{c}-\frac{1}{c-1}}$?

(A) $-2c$ (B) $2c$ (C) $c-c^2$ (D) c^2-c (E) $-c^2$

11. If the distance AB is 10 and EF is 15, find the distance CD.



(A) 9

(B) 8

(C) 7.5

(D) 6

(E) Not enough information to determine CD

12. In a Sudoku puzzle, one must place a digit (1,2,. . . ,9) in each of the 1×1 squares so that each row, each column, and each of the 3×3 grids outlined in bold contains all 9 digits. Find the digit that must go in the shaded square.

			7		9			
	3	2				5	1	
4								9
		5	3		1	8		
	1						9	
		7	9		8	2		
7								2
	5	4				1	8	
			5		4			

(A) 2 (B) 3 (C) 6 (D) 7 (E) 9

13. A factory assembly line produces a certain item, and on average 10% of the items are defective. Suppose a random sample of 10 items is taken. What is the probability that exactly 3 items in the sample are defective?

(A) $\dfrac{10!}{7!\,3!}(0.9)^7(0.1)^3$ (B) $(0.9)^7(0.1)^3$ (C) $\dfrac{10!}{7!}(0.9)^7(0.1)^3$

(D) $\dfrac{10!}{7!\,3!}(0.9)^3(0.1)^7$ (E) $(0.9)^3(0.1)^7$

14. Evaluate $(\log_2 3)\,(\log_3 4)$.

(A) 2 (B) 6 (C) $\log_2 6$

(D) $\log_6 12$ (E) none of these

15. Consider the set $S = \{1, 2, \ldots, 2004\}$. How many subsets of S are there such that the sum of their elements equals $2,009,000$?

(A) 8 (B) 10 (C) 16 (D) 1002 (E) 2^{2000}

16. If $b > 0$ and $f(x) = a + \cos^{-1}(x/b)$, find $b - a$ such that the domain and range of f are equal.

(A) π (B) $-\pi$ (C) $\pi/2$ (D) $-\pi/2$ (E) 0

17. Some of the people in a group of x people are divided into y teams of equal size. No one is on more than one team. If there are z people left over, which of the following represents the number of people on each team?

(A) $x - yz$ (B) $x/y + z$ (C) $(x - y)/z$

(D) $(x - z)/y$ (E) none of these

18. Which statement is correct?

(A) If $x^2 > 0$ then $x > 0$. (B) If $x^2 > x$ then $x > 0$. (C) If $x < 0$ then $x^2 > x$.

(D) If $x^2 > x$ then $x < 0$. (E) If $x < 1$ then $x^2 < x$.

19. What is the acute angle between the (oblique) asymptotes of the hyperbola

$$3x^2 - 7xy + 2y^2 + 3x + 4y + 2 = 0?$$

(A) 30° (B) 45° (C) 60°
(D) 75° (E) none of these

20. Statistics cited in a recent magazine revealed that the averaged (combined) weekly spending of a teenage boy and girl amounts to $\$49.04$. Of this total, on average, girls spend $\$25.75$ per week. Boys spend an average of $\$5.90$ each week on entertainment. Finally, $\$37.89$ of the amount spent each week is spent on things other than entertainment. Determine how much money boys spend each week on things other than entertainment.

(A) $\$11.49$ (B) $\$17.39$ (C) $\$22.64$
(D) $\$23.29$ (E) none of these

21. Which of these is equal to $\log_{10}\left(\dfrac{10^{2,000,000} + 10^{1,999,999}}{11 \cdot 10^{1,000,000}}\right)$?

(A) 1 (B) $999,999$ (C) $1,000,001$ (D) $1,999,999$ (E) $2,000,001$

22. If $\dfrac{a+13b}{3a-b}$ equals 3, then $\dfrac{a^3}{b^3}$ equals

(A) 64 (B) 27 (C) 8 (D) $\dfrac{127}{64}$ (E) 1

23. Five candidates—Aay, Bee, Cee, Dea, and Ewa—compete for the Wojcicka Medal in mathematics at the College of Charleston. Each candidate earns between 1 and 5 points in each of five areas—algebra, analysis, topology, discrete math, and logic—and the candidate with the most points wins the medal. There were no ties in any subject, and no ties in overall point totals. Aay came in first with 24 points, Bee came in second, Cee third, Dea fourth, and Ewa fifth. Ewa earned five points in analysis and three points in algebra.
How many points did Cee earn altogether?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

24. An interior point of an equilateral triangle is at distances 5, 7 and 8 from the three sides of the triangle. What is the common length of the sides of the triangle?

(A) It cannot be determined (B) The given configuration cannot exist
(C) 20 (D) $14\sqrt{3}$
(E) $\dfrac{40}{3}\sqrt{3}$

25. The light rays from a spotlight spread out to form a solid right circular cone, with the light rays making an angle of up to 30 degrees with the centerline of the beam. If the spotlight is 10 feet above the floor, and the centerline of the beam hits the floor at a 60-degree angle of inclination, what is the area (in square feet) that the spotlight covers on the floor?

(A) $100\pi\sqrt{3}$ (B) $25\pi\sqrt{6}$ (C) $\dfrac{100\pi}{\sqrt{3}}$ (D) $25\pi\sqrt{7}$ (E) $\dfrac{400\pi}{9}$

2019 Answers / Level 2 Test

1. B
2. D
3. B
4. A
5. C
6. D
7. E
8. E
9. D
10. E
11. D
12. E
13. A
14. A
15. B
16. A
17. D
18. C
19. B
20. B
21. B
22. C
23. C
24. E
25. B