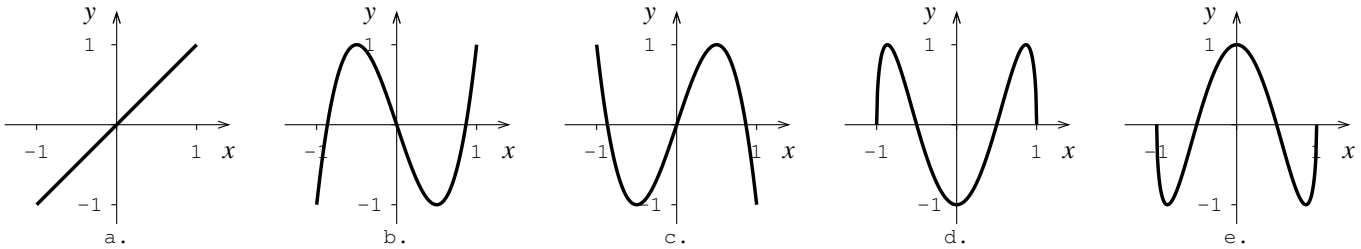


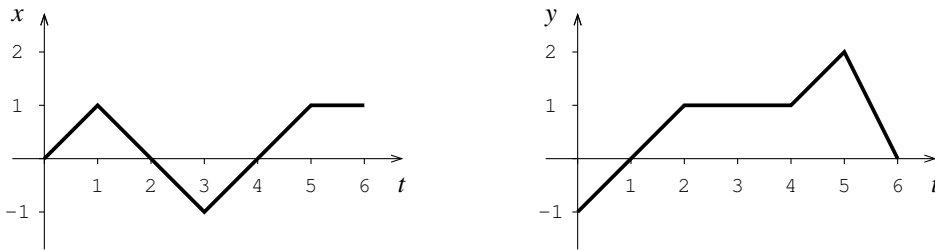
College of Charleston Math Meet 2014 Written Test – Level 3

1. Both points $(3, -1)$ and $(5, 2)$ are the same distance from the line $y = mx$, but one point lies above the line, while the other lies below the line. Find a point on the line.
 (A) $(8, 1)$ (B) $(9, 1)$ (C) $(9, 2)$ (D) $(10, 2)$ (E) $(11, 2)$
2. Find the graph of $y = \cos(3 \arccos x)$.



- (A) figure a. (B) figure b. (C) figure c. (D) figure d. (E) figure e.

3. An insect crawled about on the x - y plane for six seconds, and as it did, I plotted its x and y coordinates as functions of time t . How fast was the insect traveling at time $t = 1.5$? (x and y are measured in cm. Speeds below are in cm/sec.)



- (A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{5}$ (E) 0

4. Find the number of points of intersection between the two curves $1 + 6xy + 3y^2 = 0$ and $1 + 6xy + 3x^2 = 0$.

- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

5. Suppose that $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c , and d are real numbers. Find d if $p(1 - i) = p(-1 + i\sqrt{2}) = 0$.

- (A) -2 (B) 0 (C) 2 (D) 4 (E) 6

6. Let C be the number of gallons in a cubic foot. If a spherical fish tank has volume 36 gallons, what is its radius (in feet)?

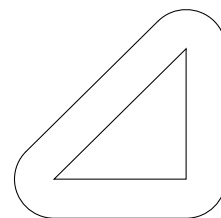
- (A) $\sqrt[3]{\frac{3}{\pi C}}$ (B) $\frac{3}{C\sqrt[3]{\pi}}$ (C) $\frac{3C}{\sqrt[3]{\pi}}$

- (D) $3\sqrt[3]{\frac{C}{\pi}}$ (E) none of these

7. Let S denote the set of all five-digit numbers in which the sum of the digits is equal to 43. Let S' be the subset of S of elements which are divisible by 11. What is the ratio of the size of the set S' to the size of the set S ?

(A) $1/3$ (B) $1/5$ (C) $1/11$ (D) $1/15$ (E) 0

8. A fort is in the shape of an isosceles right triangle, with legs of length 10m. There is a moat surrounding the fort, extending 3m out in every direction, as shown in the figure. What is the area of the moat?



(A) 78m^2 (B) 120m^2 (C) $(9\pi + 60 + 30\sqrt{2})\text{m}^2$
 (D) $(6\pi + 20 + 10\sqrt{2})\text{m}^2$ (E) none of these

9. Which of the following is equal to $\sin^6 x + 3 \sin^4 x \cos^2 x + 3 \sin^2 x \cos^4 x$?

(A) $1 - \cos^6 x$ (B) $\sin^4 x$ (C) $\cos^6 x$
 (D) $\sin^6 x$ (E) none of these

10. What is the absolute minimum value of $2x^6 + 5x^4 + x^2 + 7x^{2/3}$ on the interval $[-3, 6]$?

(A) -6 (B) 0 (C) $\sqrt{12}$
 (D) $5^{2/3}$ (E) none of these

11. Let $f(x) = (1 - x^2)^{2014}$. At $x = 0$, what is the value of the ninth derivative of f ?

(A) 2014 (B) $\frac{2014}{9!}$ (C) -2014
 (D) 0 (E) none of these

12. Suppose $g(x)$ is defined on the interval $(0, \infty)$ and $g'(x) = \frac{\sqrt{1+x^2}}{x}$. Which of the following functions could be $g(x)$?

(A) $-\sqrt{1+x^2} - \ln(1 + \sqrt{1+x^2}) + \ln x$
 (B) $-\sqrt{1+x^2} - \ln(1 + \sqrt{1+x^2}) - \ln x$
 (C) $\sqrt{1+x^2} + \ln(1 + \sqrt{1+x^2}) - \ln x$
 (D) $\sqrt{1+x^2} - \ln(1 + \sqrt{1+x^2}) + \ln x$
 (E) $\sqrt{1+x^2} + \ln(1 + \sqrt{1+x^2}) + \ln x$

13. A group of good friends go out to dinner at a restaurant and agree that they will split the cost evenly. However, by the time the bill totaling \$87.50 has arrived, two of them have left without paying. (Apparently, they were not really such good "friends" after all.) The remaining dinner guests are a little angry, but realize they only have to pay an extra \$5 each to cover the cost. How many people were in the original group?

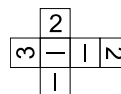
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

14. How many different tangent lines to $y = \frac{1}{x^2}$ pass through the point $(1, -4)$?
- (A) none of them pass through $(1, -4)$ (B) one does
 (C) two do (D) three do
 (E) four do

15. What is the largest prime p for which $\cos \frac{5p\pi}{6} = \sin \frac{5p\pi}{12}$?
- (A) 3 (B) 7 (C) 11
 (D) 13 (E) none of these

16. What is the sum of the solutions to $\ln(2e^{2x} - 8e^x + 10) = x$?
- (A) 4 (B) 5 (C) $\ln 4$
 (D) $\ln 5$ (E) none of these

17. The shape in the figure to the right can be folded up to form a cube. Which of the following shapes can be folded up to form a cube that is congruent to it?



- (A) (B) (C) (D) (E)

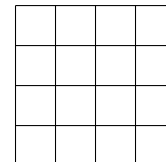
18. If we take the domain of the function $\frac{x - 3}{(x - 3)^2 + 1}$ to be $(-\infty, \infty)$, then its range is a closed interval $[a, b]$. Find $(a + b)/2$.

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

19. Five friends named Salt, Pepper, Coffee, Tea, and Vinegar exchanged gifts, and the gifts they gave were salt, pepper, coffee, tea, and vinegar. Each sent and received exactly one gift. No two sent the same present, and no one gave or received the item with the same name as herself. Salt sent coffee to Vinegar. The recipient of the vinegar sent pepper to Salt. The person with the same name as the gift sent by Coffee sent her own gift to Pepper. Who gave a gift to Coffee?

- (A) Salt (B) Pepper (C) Coffee (D) Tea (E) Vinegar

20. What proportion of the rectangles in the accompanying figure are squares?



- (A) $\frac{10}{33}$ (B) $\frac{15}{42}$ (C) $\frac{9}{25}$ (D) $\frac{3}{10}$ (E) $\frac{1}{3}$

21. Find $\lim_{n \rightarrow \infty} s(n)$ if

$$s(n) = \frac{1}{n} \left(\frac{1}{2 + \frac{1}{n}} + \frac{1}{2 + \frac{2}{n}} + \frac{1}{2 + \frac{3}{n}} + \cdots + \frac{1}{3} \right)$$

- (A) $\ln(3/2)$ (B) $\ln 6$ (C) $\ln 9$ (D) $\ln 8$ (E) $\ln 1$

22. How many real solutions are there to $\frac{1}{2x-3} + \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{2x+3} = 0$?

- (A) there are no real solutions (B) 1
(C) 2 (D) 3

(E) 4

23. How many subsets does $S = A \cup B$ have if $A = \{0, 1, -1, 2\}$ and $B = \{1, 2, 5\}$?

- (A) 8 (B) 13 (C) 32 (D) 64 (E) 7

24. Let $f(x) = \ln \left(\frac{x(x+2)^9}{(5x-1)^8} \right)$. Compute $f''(1)$.

- (A) $\frac{5}{2}$ (B) -3 (C) 0

(D) 1

(E) none of these

25. What is the remainder when the polynomial $202x^2 + 2014$ is divided by $x - 10$?

- (A) -22014 (B) 20214 (C) 22101 (D) 22214 (E) 241022

2014 Answers / Level 3 Test

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|------|-------|-------|
| 1. A | 10. B | 19. B |
| 2. B | 11. D | 20. D |
| 3. C | 12. D | 21. A |
| 4. C | 13. A | 22. D |
| 5. E | 14. B | 23. C |
| 6. E | 15. E | 24. E |
| 7. B | 16. D | 25. D |
| 8. C | 17. C | |
| 9. A | 18. C | |