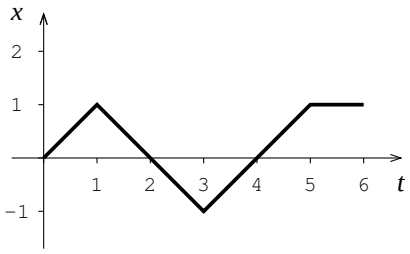
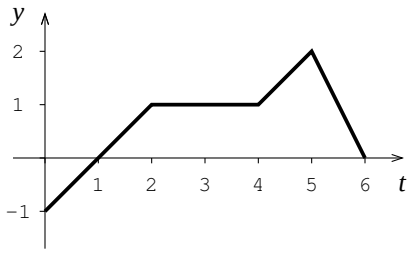
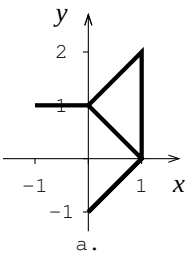
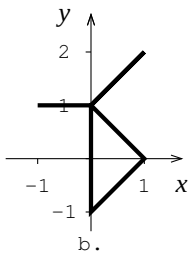
College of Charleston
Math Meet 2016
Written Test – Level 3

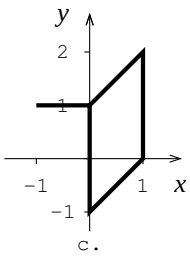
1. An insect crawled about on the x-y plane for six seconds, and as it did, I plotted its x and y coordinates as functions of time t. Which x-y graph shows the path of the insect?

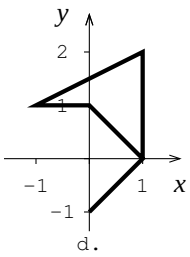


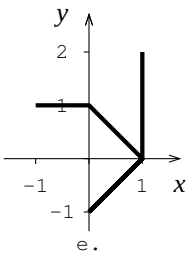












(A) figure a. (B) figure b. (C) figure c. (D) figure d. (E) figure e.

2. Both points $(2, 1)$ and $(1, 0)$ are the same distance p away from the line $y = mx$, but one point lies above the line, while the other lies below the line. Find p.

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{7}}$ (C) $\frac{1}{\sqrt{8}}$ (D) $\frac{1}{3}$ (E) $\frac{1}{\sqrt{10}}$

3. How many different numbers can be written in the form $x + y$ where x and y are in $\{1, 2, 4, \ldots, 2^{11}\}$?

(A) 21 (B) 32 (C) 66 (D) 78 (E) 121

4. The solution set to the equation $x^{2016} = 2016^x$ consists of

(A) one positive number only.

(B) one negative number only.

(C) two positive numbers only.

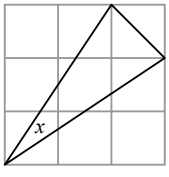
(D) one positive number and one negative number only.

(E) two positive numbers and one negative number only.

5. The curve $y = x + \dfrac{3}{x+1}$ is symmetric through the point (h, k). Find $h + 3k$.

(A) -4 (B) 3 (C) -2 (D) 1 (E) 0

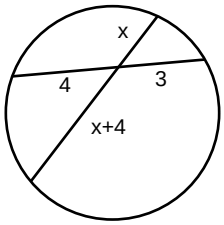
6. Let x be the angle at (0,0) in the triangle with vertices (0,0), (3,2), and (2,3). Find $\sin x$.



(A) 3/13 (B) 7/26 (C) 4/13 (D) 9/26 (E) 5/13

7. As an elevator moves up and down in a vertical shaft, we measure its altitude in meters above the ground floor as a function of time in seconds. Suppose that the elevator's acceleration is a constant 5 m/sec^2, and that its position at time $t = 1$ is 2 meters higher than its position at time $t = 0$. What was the elevator's velocity at time 0?
(Answers are in m/sec.)
(A) $-1/2$ (B) $-1/4$ (C) 0 (D) $1/4$ (E) $1/2$

8. A *chord* is a line segment whose endpoints lie on a circle.
Two chords lie in the same circle. If the first divides the second into lengths 3 and 4, and the second divides the first into lengths x and x+4, find x.



(drawing not to scale)

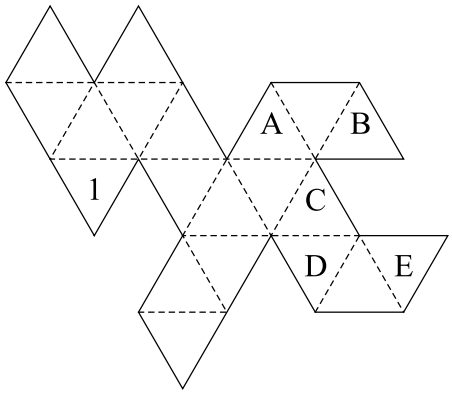
(A) 2 (B) $-1 + \sqrt{13}$ (C) $2 + \sqrt{5}$ (D) $3 - \sqrt{5}$ (E) $5 - \sqrt{2}$

9. A jar is filled with two types of candies: lemon drops and mint drops. Each piece of candy is wrapped either in red paper or white paper.
7/10 of the candy is in red paper.
3/10 of the candy consist of mint drops wrapped in red paper.
3/5 of the candy is lemon drops.
If you pull a mint drop from the jar, what's the probability that it is wrapped in white paper?
(A) 1 (B) 4/5 (C) 1/2 (D) 2/5 (E) 1/4

10. Evaluate the limit:
$$\lim_{h \to 0} \frac{\cos(x - \sin h) - \cos x}{\cos x \sin h}$$
(A) $\sin x$ (B) $\cos x$ (C) $\tan x$ (D) $\sec x$ (E) 1

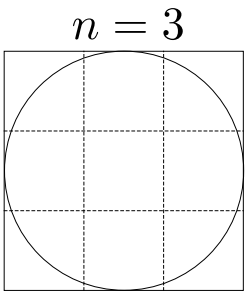
11. Let $a_n = \sin\left(\frac{n\pi}{6}\right)$. What is the average of the numbers $a_1, \ldots, a_{100}$?
(A) $\frac{\sqrt{3}}{2} + \frac{3}{2}$ (B) $\frac{1}{2\sqrt{3}} + \frac{1}{2}$ (C) $\frac{2}{5} + \frac{\sqrt{3}}{5}$
(D) $\frac{3}{200} + \frac{\sqrt{3}}{100}$ (E) none of these

12. The figure to the right could be folded up to form an icosahedron. Which face would be opposite the face labeled 1?



(A) A (B) B (C) C (D) D (E) E

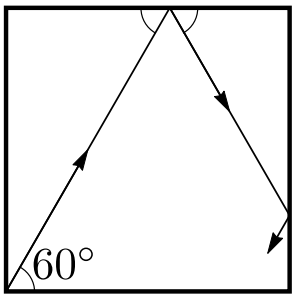
13. Start with a big square S. Using equally spaced lines parallel to its sides, divide it into n^2 congruent small squares. If we inscribe a circle in S, what's the largest value of n for which each of these small squares includes a point inside the circle?



(A) 5 (B) 6 (C) 7

(D) 8 (E) none of these

14. A beam of light leaves a vertex of a unit square at an angle of 60° to one of the sides. Every time the beam of light hits one of the sides of the square, it reflects back into the square. What distance will the beam of light travel before it next hits a vertex?



(A) $\sqrt{5}$ (B) $2\sqrt{3}$

(C) 4 (D) $\sqrt{17}$

(E) It will never again hit a vertex.

15. If $p(x)$ is a polynomial, and if $p(x) + ip(1-x) = \sqrt{2}x^5$, what's the coefficient of x^5 when $p(x)$ is written in the standard power form $p(x) = a + bx + cx^2 + \cdots$? (Here, i denotes $\sqrt{-1}$.)

(A) $1 - i$ (B) $\sqrt{2}(1-i)$ (C) $\sqrt{2}(1+i)$ (D) $\dfrac{1-i}{\sqrt{2}}$ (E) $\dfrac{1+i}{\sqrt{2}}$

16. Given that $a + \ln a = 0$, what is the solution to $x^4 e^x = 256$?

(A) $x = 4a$ (B) $x = a + 4$ (C) $x = \sqrt[4]{256a}$

(D) $x = a + \ln 256$ (E) none of these

17. For a function $f(x)$, let $L(f) = \dfrac{1}{x}\dfrac{df}{dx}$. What is $L(L(f))$?

(A) $-\dfrac{1}{x^3}f''(x)$ (B) $\dfrac{1}{x}f''(x)$ (C) $\dfrac{1}{x^3}(xf''(x) - f'(x))$

(D) $\dfrac{1}{x^2}f''(x)$ (E) none of these

18. Let $f(x) = \left(1 + \tan^2 x\right)^n$. What is $f''(\dfrac{\pi}{4})$?

(A) $n(2n-1)2^n$ (B) $n^2 2^{n+3}$ (C) $n(n+1)2^{n+2}$

(D) $n(n-1)2^{n-2}$ (E) none of these

19. Simplify $1 + 5(x-1) + 10(x-1)^2 + 10(x-1)^3 + 5(x-1)^4 + (x-1)^5$.

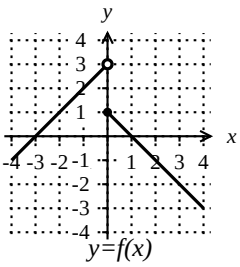
(A) $x^5 - x^3 + 1$ (B) $x^5 - x^3 + x^2$ (C) $x^5 + x^3 - x^2$

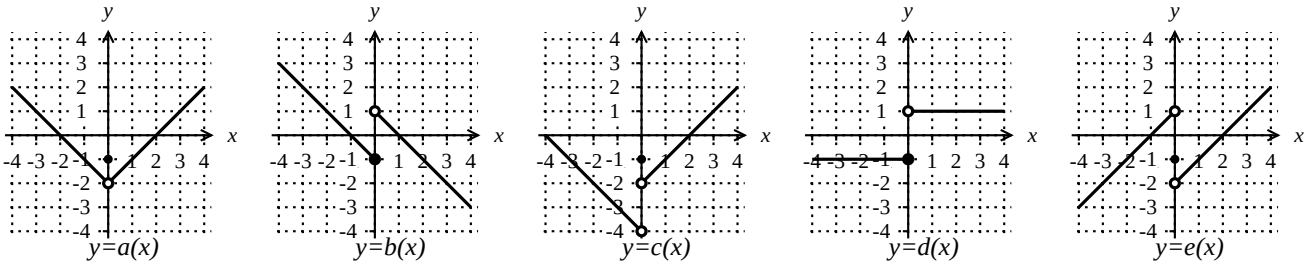
(D) x^5 (E) none of these

20. If a, b, c, d, e, and f are the functions graphed below, how many of the five functions

$$f(x) + a(x), \quad f(x) + b(x), \quad f(x) + c(x), \quad f(x) + d(x), \quad f(x) + e(x)$$

are odd?





(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

21. If $\sin x = \dfrac{1}{4}$, what is $\cos(4x)$?

(A) $\dfrac{1}{256}$ (B) $\dfrac{\sqrt{15}}{64}$ (C) $\dfrac{225}{256}$

(D) 1 (E) none of these

22. What is the equation of the tangent line to

$$y = (1 + \sin x)(1 + \sin(2x))(1 + \sin(3x)) \cdots (1 + \sin(100x))$$

at the point $(0, 1)$?

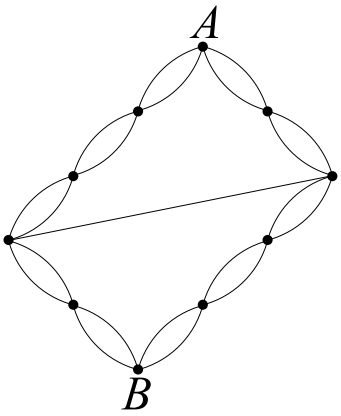
(A) $y = 1$ (B) $y = 1 + 100!x$ (C) $y = 1 + 100x$

(D) $y = 1 + 5050x$ (E) none of these

23. Six students are going to stand in line to get tickets to a concert. How many ways are there for the students to be arranged in the line if Betsy must stand immediately behind Sarah?

(A) 5 (B) 120 (C) 320 (D) 720

24. In the picture to the right, each dot represents a city, and each path (curved or straight) connecting dots represents a way to get directly from one city to another. If we impose the restriction that we may visit a city at most once, how many different ways are there to get from city A to city B?



(A) 4 (B) 64 (C) 144

(D) 2048 (E) none of these

25. Which of the following statements is logically equivalent to "If Wendy gets a high score on the written test, then Wendy knows her math"?

(A) Wendy doesn't know her math unless she gets a high score on the written test.

(B) If Wendy knows her math, then she gets a high score on the written test.

(C) Wendy knows her math only if she gets a high score on the written test.

(D) If Wendy doesn't know her math, she can't get a high score on the written test.

(E) Wendy doesn't get a high score on the written test only if she doesn't know her math.

2016 Answers / Level 3 Test

1. A
2. E
3. D
4. E
5. A
6. E
7. A
8. A
9. E
10. C
11. D
12. B
13. B
14. E
15. E
16. A
17. C
18. C
19. D
20. B
21. E
22. D
23. B
24. C
25. D