College of Charleston
Math Meet 2017
Written Test – Level 3

1. If $\dfrac{1}{x} + y = 2$ and $\dfrac{1}{y} + x = 3$, what's $\dfrac{y}{x}$?

   (A) $\dfrac{\sqrt{3}}{1-\sqrt{3}}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

   (D) $\dfrac{-\sqrt{3}}{1+\sqrt{3}}$ (E) none of these

2. If $\{a_0, a_1, a_2, \ldots\}$ is a sequence of numbers, if

   $$a_{n+2} - 2a_{n+1} + a_n = 0 \quad \text{for all } n = 0, 1, 2, \ldots$$

   and if

   $$a_{1000} = 102 = a_{996} - 1$$

   find the tens digit of a_0.

   (A) 9 (B) 7 (C) 5 (D) 3 (E) 1

3. Which of these is an asymptote of $3x^2 + 17xy - 28y^2 = 2$?

   (A) $x - 7y = 0$ (B) $x + 2y = 0$ (C) $3x + 2y = 0$

   (D) $3x - 4y = 0$ (E) $2x - 3y = 0$

4. Find $g'(0)$ if $g(0) = 1$ and

   $$\frac{g(x)}{x^2(x+1)^3} = \frac{A}{x^2} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}.$$

   (A) 0 (B) 1 (C) -2 (D) 3 (E) -4

5. Evaluate the limit $\displaystyle\lim_{h\to 0} \frac{e^{x^2+h} - e^{(x+h)^2}}{h}$

   (A) 0 (B) $(x^2-1)e^{x^2}$ (C) $(1-2x)e^{x^2}$

   (D) $2xe^{x^2}$ (E) $(x^2+1)e^{x^2}$

6. Find the area of the region bounded by the curves $y = 2x^2$ and $y - 4x = 6$.

   (A) $76/3$ (B) $80/3$ (C) $68/3$ (D) $32/3$ (E) $64/3$

7. Find z if x, y, $(z-x)$ and $(z-y)$ are positive real numbers satisfying

   $$\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + (z-y)^2 &= 4 \\ (z-x)^2 + y^2 &= 6 \end{aligned}$$

   (A) $\sqrt{5+2\sqrt{2}}$ (B) $\sqrt{5-2\sqrt{2}}$ (C) $\sqrt{5+3\sqrt{2}}$ (D) $\sqrt{5-3\sqrt{2}}$ (E) $\sqrt{5+\sqrt{2}}$

8. The CDC recently ran a program to immunize children against cooties. 1.6 million children were immunized over the 100-day program and their age, height, and weight (rounding to the nearest integer) were recorded. If the ranges of this data were

	minimum	*maximum*
Age (months)	13	142
Height (cm)	31	150
Weight (kg)	1	110

then how many of the following statements must be true? (In these statements, "age," "height," and "weight" refer to the child's *recorded* age, height, and weight.)

I. At least one child was immunized every day of the program.
II. One day during the program, two children with the same age, height, and weight were immunized.
III. One day during the program, two children with the same age and height were immunized.
IV. One day during the program, two children with the same height and weight were immunized.
V. One day during the program, two children with the same age and weight were immunized.
VI. Two children with the same age, height, and weight were immunized during the program.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. For $x > 0$ one can write $\left(\dfrac{x^{7/2}}{x^{2/3}}\right)^{-6}$ as

(A) x^{17} (B) 1 (C) x^{25}

(D) $\dfrac{1}{x^{17}}$ (E) none of these

10. Find the sum of all real roots of $x^4 - x^2 - 20 = 0$.

(A) 0 (B) 1 (C) 4 (D) 5 (E) 9

11. The ancient Mayans were unusual in that they used a *vegesimal* number system. This means that it was very much like our number system, but base-twenty rather than base-ten. They also wrote their digits in a vertical column and their symbol for zero looked sort of like an empty basket ($\ominus$). Which of these choices was their way of writing the number 2005?

(A) $\begin{array}{c} \cdot\cdot \\ \ominus \\ - \end{array}$ (B) $\begin{array}{c} - \\ \ominus \\ - \end{array}$ (C) $\begin{array}{c} \sim \\ \ominus \\ \ominus \\ \sim \end{array}$ (D) $\begin{array}{c} \triangleright \\ \ominus \\ \ominus \\ - \end{array}$ (E) $\begin{array}{c} \triangleright \\ \ominus \\ \ominus \\ \cdot\cdot \end{array}$

12. The derivative of $f(x) = \ln(x \ln x)$ is

(A) $\dfrac{1 + \ln x}{x \ln x}$ (B) $\dfrac{1}{x^2}$ (C) $\dfrac{\ln x}{x}$ (D) $\dfrac{1}{2x}$ (E) $\dfrac{1}{x \ln x}$

13. How many three-digit numbers are divisible by their final (rightmost) digit?

(A) 100 (B) 319 (C) 329

(D) 420 (E) none of these

14. For any positive integer n, the function $\tau(n)$ is defined to be the number of factors of n. For example, $\tau(10) = 4$ because 10 has 4 factors: $1, 2, 5$, and 10. What must be true if $\tau(m)$ is an odd number?

(A) m is odd (B) m is a perfect square (C) m is a power of 3 (D) m is prime (E) $\tau(m)$ is 3

15. Which of these functions is in the form $f(x) = a + \log_3(x - c)$, satisfies $f(11) = 10$ and has a graph with $x = 2$ as an asymptote?

(A) $f(x) = 10 + \log_3(x + 2)$ (B) $f(x) = 11 + \log_3(x - 1)$ (C) $f(x) = 8 + \log_3(x - 2)$ (D) $f(x) = 12 - \log_3(x - 2)$ (E) None of the Above

16. Levin and Alex are playing the following coin-toss game. They take turns tossing the coin, beginning with Levin, and the first player to get tails loses the game.
Assuming that the two outcomes of a coin toss—heads and tails—occur with equal probability, what's the probability that Levin will win the game?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) $\frac{4}{7}$

17. For which of the following values of θ will

$$x^2 - (2\cos\theta)x + 1$$

be a factor of

$$x^{12} + x^{10} + x^8 + x^6 + x^4 + x^2 + 1?$$

(A) $2\pi/7$ (B) $3\pi/8$ (C) $4\pi/9$ (D) $\pi/2$ (E) $6\pi/11$

18. If $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then ω^3 and ω^6 are, respectively

(A) 1 and 1 (B) 1 and -1 (C) -1 and 1 (D) -i and -1 (E) i and -1

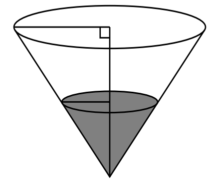
19. Right now, there is a room here at the College where nineteen students are taking this test. Eleven of them are from South Carolina. Ten of the students are female. Only two of the male students in the room are from *outside* South Carolina. How many of the students in the room are female and from South Carolina?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

20. One person computes the average of a list of 100 test scores and writes it at the end of the list. Call this number a_1. Another person finds the average of those 101 numbers (the test scores *and* their average a_1). Call this second average a_2. If for those 100 test scores, the average a_1 was less than the median score m, what must be true?

(A) $a_1 > a_2$ (B) $a_2 > a_1$ (C) $a_1 = a_2$ (D) $m < a_2$ (E) none of the above

21. A tank in the shape of an inverted cone is partially filled with water. When more water is added, the height of the water in the tank increases by 10 cm, and the volume of water in the tank doubles. Find the original height (in cm) of water in the tank.



(A) $\dfrac{10}{\sqrt[3]{2}-1}$ (B) $\dfrac{10}{\sqrt[3]{2}}$ (C) $5\sqrt[3]{4}$

(D) $\dfrac{5}{\sqrt[3]{2}+1}$ (E) Not enough information.

22. Three circles pass through the origin of a Cartesian plane. The center of the first circle belongs to the first quadrant, the center of the second circle belongs to the second quadrant, and the center of the third circle belongs to the third quadrant. Let P be the intersection of the interiors of the three circles. Find the true statement.

(A) P must be the empty set.

(B) P may be nonempty, in which case it must be a subset of quadrant one.

(C) P may be nonempty, in which case it must be a subset of quadrant two.

(D) P may be nonempty, in which case it must be a subset of quadrant one or three.

(E) P may be nonempty, in which case it must be a subset of quadrant four.

23. The three sides of a triangle have lengths 2, 3, and 4. Find the area of the triangle.

(A) $\dfrac{\sqrt{90}}{10}$

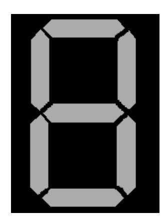
(B) $\dfrac{3\sqrt{15}}{4}$

(C) $\dfrac{9}{2}$

(D) $3\sqrt{3}$

(E) Not enough information is given to determine the area.

24. Each letter or digit on the front panel of a microwave oven is comprised of seven individual LED's in the configuration shown. Each LED may be activated (on) or not (off) according to what letter or digit the configuration is to represent. When activated, an LED glows brightly and forms an individual segment of a letter or digit.



A power surge causes the individual LED's to be activated in a random fashion with each segment having a probability of $.5$ of being activated. Find the probability that the configuration of the LED's that are activated causes an even digit to appear on the panel.

(A) $\dfrac{1}{2^5}$ (B) $\dfrac{1}{10^7}$ (C) $\dfrac{5}{10^7}$

(D) $\dfrac{5}{2^7}$ (E) none of these

25. If $x = 100!$, $y = 100!/60!$, and $z = 100^{40}$, find the true statement.
(A) $x < y < z$ (B) $y < x < z$ (C) $y < z < x$ (D) $z < x < y$ (E) $z < y < x$

2017 Answers / Level 3 Test

1. B
2. C
3. D
4. D
5. C
6. E
7. A
8. D
9. D
10. A
11. B
12. A
13. D
14. B
15. C
16. B
17. A
18. A
19. E
20. C
21. A
22. C
23. B
24. D
25. C