

**College of Charleston
Math Meet 2018
Written Test – Level 3**

1. I chose two numbers a and b and then defined the rule L that assigns to each polynomial $p(x)$ the number

$$L(p(x)) = ap(1) + bp(2)$$

I then tested my rule on the monomials

$$1, x, x^2, x^3, x^4, \dots$$

and recorded the results.

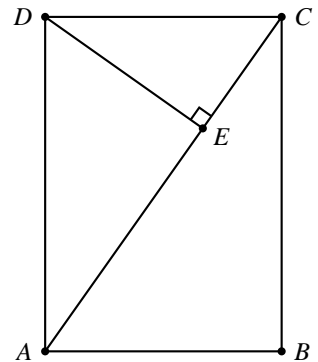
For instance, if $a = 1$ and $b = -2$, then

$$L(1) = -1, \quad L(x) = -3, \quad L(x^2) = -7, \dots$$

Find the sequence which **cannot** be

$$L(1), L(x), L(x^2), L(x^3), L(x^4), L(x^5), \dots$$

- (A) $2, 1, -1, -5, -13, \dots$ (B) $1, 2, 4, 8, 16, \dots$
 (C) $2, -2, -10, -20, -35, \dots$ (D) $1, 3, 7, 15, 31, \dots$
 (E) $2, -1, -7, -19, -43, \dots$
2. Find the area of the rectangle $ABCD$ if (the length) $\overline{EC} = 8$, and $\overline{AE} = \overline{ED} \times \sqrt{2}$.
 (Figure not to scale.)



- (A) $192\sqrt{2}$ (B) $196\sqrt{2}$ (C) $198\sqrt{2}$ (D) $188\sqrt{2}$ (E) $164\sqrt{2}$
3. A teacher tried to divide a bag of pennies among her 3 favorite students, but after she gave each of them the same number of pennies, there was one penny left over. She then tried to divide the pennies equally among her 5 favorite students, but there was a penny left over, then among her 7 favorites, but there was a penny left over. Assuming she had more than one penny, what's the fewest number of pennies she could have had in her bag? The first (leftmost) digit in the correct answer is
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. Claire is a little dizzy from too much sun at the beach and she starts walking in a strange way:
- she takes one step forward,
 - she turns 90° to her right and then takes two steps forward,
 - she turns 90° to her right and then takes one step forward,
 - she turns 90° to her left and then takes one step backward,
 - she starts all over again.

Each step is 1 yard. After 186 steps, Claire passes out. How many yards from where she started does Claire end up?

- (A) 186 (B) 1 (C) 2 (D) $\sqrt{2}$ (E) $\sqrt{5}$

5. Farmer Mary keeps four kinds of animals: cows, chickens, ducks, and pigs. Next year, she would like to have 14 cows and chickens (that is, her cows and chickens should total 14), 12 chickens and ducks, 8 ducks and pigs, and 10 pigs and cows. What's the total number of animals must she have altogether?

- (A) 22
 (B) 32
 (C) 42
 (D) It's impossible for her to do this.
 (E) It's possible for her to do this, but there's more than one possible total.

6. When the polynomial $p(x)$ is divided by $x^2 - 1$, the remainder is $x + 2$. When $p(x)$ is divided by $x^2 - 4$, the remainder is $x + 1$. Find the remainder when $p(x)$ is divided by $(x - 1)(x - 2)$.

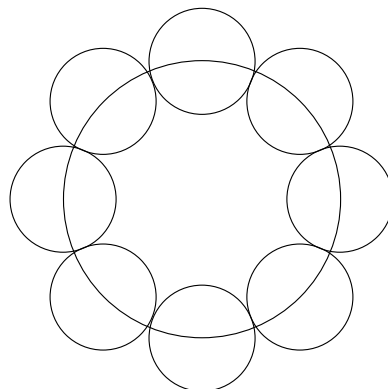
- (A) $(x + 1)(x + 2)$ (B) $x + 1$ (C) $x - 1$
 (D) $x + 2$ (E) 3

7. Find $A + B + C$ so that

$$\frac{\sin(5x)}{\sin x} = A \cos(4x) + B \cos(2x) + C$$

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. C is a circle. Eight circles having radius $\frac{1}{2}$ are arranged so their centers all lie on the circumference of C and so each one intersects exactly two others, tangentially. What is the area of C ?



- (A) 2π (B) $4\pi\sqrt{2}$ (C) 8π
 (D) $(4 + 2\sqrt{2})\pi$ (E) none of these

9. Two opposite edges of a tetrahedron are perpendicular, have lengths 7 units and 8 units respectively, and the distance between them (as measured along a line segment perpendicular to both edges) is 6 units. In cubic units, what is the volume of the tetrahedron?

- (A) 56 (B) 108
(C) 168 (D) 336

(E) not enough information to tell

10. Find the domain of the function

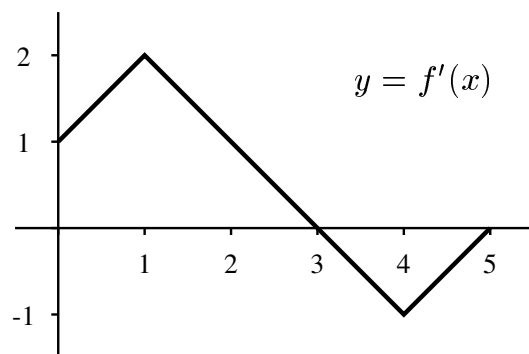
$$\sqrt{\frac{(x+3)(x-1)}{x+1}}$$

- (A) $[-3, \infty)$ (B) $[-3, -1) \cup [1, \infty)$ (C) $(-\infty, -3] \cup [1, \infty)$
(D) $(-\infty, -3] \cup (-1, 1]$ (E) $[-3, -1) \cup (-1, 1]$

11. The 2003 inhabitants of an island are divided in two groups: the "truth tellers", who always tell the truth, and the "liars", who always lie. Each person is exactly one of the following: a singer, a soccer player or a fisherman. We ask each inhabitant the following three questions: 1) Are you a singer? 2) Are you a soccer player? 3) Are you a fisherman? 1000 people answer "yes" to the first question, 700 people answer "yes" to the second question, 500 people answer "yes" to the third question. How many "liars" are there on the island?

- (A) 105 (B) 183 (C) 197 (D) 319 (E) 732

12. The graph of $f'(x)$ appears at right. If $f(0) = 2$, what is $f(5)$?



- (A) -1.5 (B) 0 (C) 2.5 (D) 4.5 (E) 5.5

13. The Yankees and the Red Sox are having a playoff series to determine the American League champion. The series continues until one team has won three games, and so may take up to five games. Assuming each team has an equal chance to win each game, what is the expected number of games in the series?

- (A) $\frac{17}{4}$ (B) $\frac{15}{4}$ (C) $\frac{25}{8}$ (D) $\frac{33}{8}$ (E) 4

14. If Paul has fish for dinner, then he'll either pay his rent or read a book (or both). If he pays his rent, then he'll sleep in on Monday. If he does not read a book, then he'll not sleep in on Monday. Paul does not sleep in on Monday. What can you conclude?
- (A) Paul paid his rent.
 (B) If he does not have fish for dinner, then he'll read a book.
 (C) If he pays his rent, then he'll *not* have fish for dinner.
 (D) If he reads a book, then he'll sleep late on Monday.
 (E) The sentence in answer d. is false.

15. Figure 1 shows a map of eight cities, A,B,C,D,E,F,G,H, and routes for ten possible roads between the cities. A contractor wants to build as few of these roads as necessary so that it will be possible to go from any of the eight cities to any other using the finished roads. See, for example, Figure 2. How many ways are there to do this?

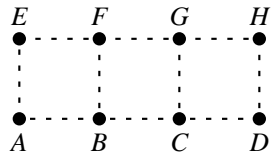


Fig. 1

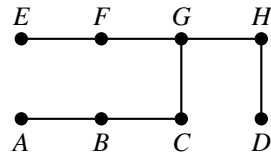


Fig. 2

- (A) 120 (B) 69 (C) 56 (D) 48 (E) 35

16. Find $\frac{dy}{dx}$ if $y = \ln(\sqrt{x^2 + 9} - 3) - 2 \ln \sqrt{x}$.

(A) $\frac{-3}{x\sqrt{x^2 + 9}}$

(B) $\frac{-x}{\sqrt{x^2 + 9}(\sqrt{x^2 + 9} + 3)} + \frac{1}{x}$

(C) $\frac{-9}{x(x^2 + 9)}$

(D) $\frac{3}{\sqrt{x}(x^2 + 9)}$

(E) $\frac{3 - x + \sqrt{x^2 + 9}}{x^2}$

17. $\cos^{-1}\left(\sin \frac{7\pi}{5}\right) =$

(A) $\frac{\pi}{10}$

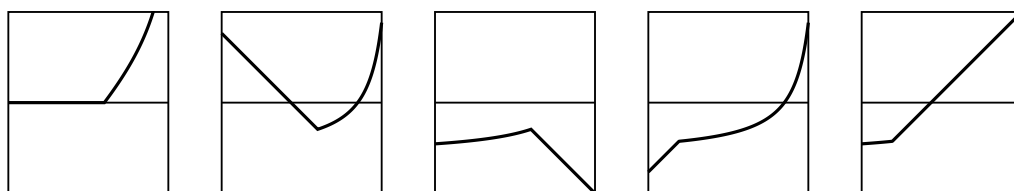
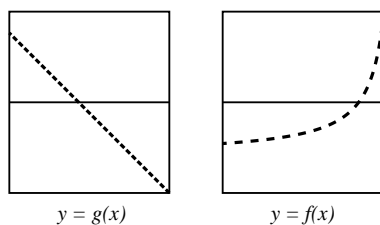
(B) $\frac{5\pi}{10}$

(C) $\frac{7\pi}{10}$

(D) $\frac{9\pi}{10}$

(E) $\frac{11\pi}{10}$

18. I've graphed two functions f and g in windows that show part of the x -axis but not the y -axis. Beneath these I graphed five other functions. Find the function that I did *not* graph. (All seven graphs use the same window settings, that is, the same minimum and maximum values of x and of y .)



- (A) $\frac{f + g - |f + g|}{2}$ (B) $\frac{f - g + |f - g|}{2}$ (C) $\frac{f + g - |f - g|}{2}$
 (D) $\frac{f + g + |f - g|}{2}$ (E) $\frac{f - g - |f + g|}{2}$

19. What is the smallest positive θ such that both $\cot \theta$ and $\csc \theta$ are primes?

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{5}$
 (D) $\frac{\pi}{7}$ (E) There are no such values

20. Let $i = \sqrt{-1}$. Then $(1 - 2i)^4 =$

- (A) $-1 + 30i$ (B) $-3 + 28i$ (C) $-5 + 26i$ (D) $-7 + 24i$ (E) $-9 + 22i$

21. If A and B are subsets of the set U , then

- $A \cap B$ is the set of all elements that belong to both A **and** B ,
- $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
- A' is the set of all elements of U that are **not** in A .

If A , B , and C are subsets of U . Which of these equals

$$(A' \cup B) \cap (B' \cup C) \cap (A \cup C)?$$

- (A) $(A \cup C)' \cup (B \cap C)$ (B) $(A \cap C') \cup (B \cup C)'$ (C) $(A \cup C)' \cup (B \cap C')$
 (D) $(A \cap C) \cup (B \cup C)'$ (E) $(A \cup C)' \cup (B \cap C)$

22. Simplify $(\sqrt{2} - 1)^1 + (\sqrt{2} - 1)^2 + (\sqrt{2} - 1)^3 + \dots + (\sqrt{2} - 1)^{2015}$.
- (A) $\frac{1 - (\sqrt{2} - 1)^{2015}}{\sqrt{2}}$ (B) $(\sqrt{2} - 1)^{2016}$ (C) $\sqrt{2}(\sqrt{2} - 1)^{2015}$
 (D) 2^{1008} (E) none of these
23. Find the ternary (base 3) expansion of $\frac{1}{4}$.
- (A) 0.02002002... (B) 0.020102010201... (C) 0.02020202...
 (D) 0.0201010101... (E) 0.0200100200100...
24. Ship A and Ship B leave the same port at the same time. Ship A travels in a straight line at 10 miles per hour. Ship B travels in a straight line 15 miles per hour. How fast is the distance between the ships increasing if their routes form at a 60° angle?
 Answers below are in miles per hour. Assume for this problem that the Earth is flat.
- (A) $5\sqrt{5}$ (B) $5\sqrt{7}$ (C) $5\sqrt{19}$
 (D) $5\sqrt{13 - 6\sqrt{3}}$ (E) $5\sqrt{13 + 6\sqrt{3}}$
25. Amazingly, there is an interesting relationship between the function

$$G(x) = \frac{x}{1 - x - x^2}$$

and the famous Fibonacci Sequence: $F_0 = 0, F_1 = 1, F_{n+1} = F_n + F_{n-1}$ for $n > 1$. It turns out that for $0 \leq x < 1$ the output of the function G can be determined using the infinite sum

$$G(x) = F_0 + F_1x + F_2x^2 + F_3x^3 + \dots$$

Which of these is the start of the decimal expansion of $G(1/10)$?

- (A) .0011235855... (B) .0112358550... (C) .012359550...
 (D) .1123585505... (E) .1123595505...

2018 Answers / Level 3 Test

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|------|-------|-------|
| 1. C | 10. B | 19. E |
| 2. A | 11. C | 20. D |
| 3. A | 12. D | 21. E |
| 4. C | 13. D | 22. A |
| 5. A | 14. C | 23. C |
| 6. E | 15. C | 24. B |
| 7. E | 16. B | 25. E |
| 8. D | 17. D | |
| 9. A | 18. A | |