C of C Math Meet 2004
MARATHON

Rules:

i The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.

ii Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.

iii All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.

iv The judges' decisions will be final.

v All papers are to be mailed to the following address or submitted electronically to mathmeet@cofc.edu

Math Meet
MATH MARATHON
Department of Mathematics
College of Charleston
Charleston, SC 29424

vi The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verication

vii All entries must be postmarked no later than the date fees are due.

1. Find all ways to express 2004 as the sum of three integer squares.

2. Once a bright young lady named Jillian
Summed the numbers from one to a billion.
But it gave her the fidgets
To add up their digits;
If you help her, she'll thank you a million.

3. A rectangle is inscribed in a circle of unit radius. The rectangle has area equal to half the area of the circle. What is the perimeter of the rectangle?

4. A sequence of real numbers $a_1, a_2, a_3, \ldots$ is defined by $a_1 = A$, $a_2 = B$, and $a_n = Ca_{n-1} + Da_{n-2}$ for all $n > 2$. Determine all possible conditions on the real constants A, B, C and D so that the sequence $\{a_n\}$ is a geometric sequence (geometric progression).

5. A sequence of real numbers $a_1, a_2, a_3, \ldots$ is defined by $a_1 = A$, $a_2 = B$, and $a_n = Ca_{n-1} + Da_{n-2}$ for all $n > 2$. Determine all possible conditions on the real constants A, B, C and D so that the sequence $\{a_n\}$ is an arithmetic sequence (arithmetic progression).