

C of C Math Meet 2005 MARATHON

Rules:

- i. The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- ii. Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- iii. All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- iv. The judges' decisions will be final.
- v. All papers are to be mailed to the following address or submitted electronically to mathmeet@cofc.edu

Math Meet
MATH MARATHON
Department of Mathematics
College of Charleston
Charleston, SC 29424

- vi. The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification .
- vii. All entries must be received or postmarked by February 19, 2005.

1. Among the first 2005 positive integers, which has the largest number of divisors?
2. A rectangle is inscribed in a circle of unit radius. The perimeter of the rectangle is equal to three-fourths of the circumference of the circle. Find the area of the rectangle.
3. Alex leaves his house on his dirt bike. He rides north for one mile, then he turns and rides west for half a mile, then he turns south and rides for a quarter mile, then east for an eighth of a mile, then north again for a sixteenth of a mile, west for $1/32$ of a mile, and so on. Eventually he reaches a point where he is just spinning in a counterclockwise circle in one spot. How far is Alex from his house?
4. Find the smallest positive integer n which has the property that the product of any set of n consecutive integers is divisible by 2005.
5. Consider the sequence of numbers which begins 6,3,... and has the property that each successive term in the sequence is equal to the average of the two terms immediately preceding it. Find the sum of the first 1000 terms of this sequence, accurate to ten decimal places.