

**College of Charleston**  
**Math Meet 2005**  
**Written Test – Level 2**

1. A coin is biased so that the probabilities of heads and tails are  $\frac{2}{3}$  and  $\frac{1}{3}$  respectively. A second coin is biased so that the probabilities of heads and tails are  $\frac{3}{5}$  and  $\frac{2}{5}$  respectively. If both coins are tossed, find the probability of at least one head.

- (A)  $\frac{13}{15}$  (B)  $\frac{2}{5}$   
(C)  $\frac{8}{15}$  (D)  $\frac{3}{5}$

(E) None of the above

2. A pharmacist has 8 liters of a 15 percent solution of acid. How much distilled water must she add to reduce the concentration of acid to 10 percent?

- (A) 4 liters (B) 5 liters  
(C) 6 liters (D) 7 liters

(E) None of the above

3. Solve for  $x$ :

$$3 \leq |x + 2| < 8$$

- (A)  $-10 < x \leq -1$  or  $5 \leq x < 6$ .  
(B)  $-10 < x \leq 1$  or  $-5 \leq x < 6$ .  
(C)  $-10 < x \leq 5$  or  $-1 \leq x < 6$ .  
(D)  $-10 < x \leq -5$  or  $1 \leq x < 6$ .  
(E)  $-10 < x \leq -5$  or  $-1 \leq x < 6$ .

4. You are dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. (Half the cards in a deck are red). Find the probability that both cards dealt are red.

- (A)  $\frac{25}{102}$  (B)  $\frac{25}{51}$  (C)  $\frac{1}{2652}$

(D)  $\frac{13}{51}$  (E) None of these

5. Evaluate  $e^{3 \ln 0.5} + \ln \sqrt[3]{e}$ .

- (A)  $\frac{13}{2}$  (B)  $\frac{1}{2}$   
(C)  $\frac{11}{24}$  (D)  $\frac{23}{24}$

(E) None of the above

6. Find all values of  $x$  in  $[0, 2\pi]$  satisfying

$$\cot x + \sqrt{3} = 0$$

- (A)  $\frac{2\pi}{3}, \frac{4\pi}{3}$  (B)  $\frac{2\pi}{3}, \frac{5\pi}{3}$   
 (C)  $\frac{5\pi}{6}, \frac{11\pi}{6}$  (D)  $\frac{5\pi}{6}, \frac{7\pi}{6}$   
 (E) None of the above

7. Solve the following equation for  $x$

$$\log_2(x - 2) + \log_2(x) = 3.$$

How many solutions are there?

- (A) 0 (B) 1  
 (C) 2 (D) 3  
 (E) None of the above

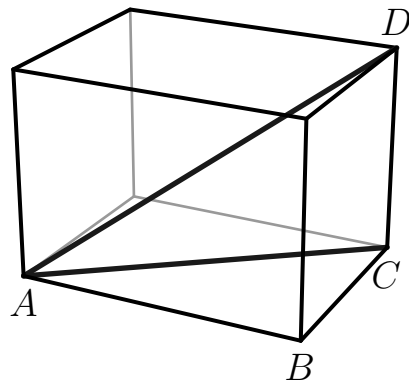
8. Write the complex number  $\frac{1 - 3i}{1 + 3i}$  in the form  $a + bi$  where  $a$  and  $b$  are real numbers.

- (A)  $1 + 6i$  (B)  $-0.8 - 0.6i$   
 (C)  $8 - 6i$  (D)  $0.8 + 0.6i$   
 (E) None of the above

9. If  $\alpha = \log_5(3)$  and  $k^\alpha = 125$  then what is  $k^{(\alpha^2)}$ ?

- (A)  $\log_5(30)$  (B) 9 (C) 15  
 (D) 27 (E)  $3^{\log_5(30)}$

10. The figure shows a rectangular solid with length  $AB = 4$ , width  $BC = 3$ , and height  $CD = 5\sqrt{3}/3$ . Find the angle  $CAD$ .



- (A)  $\frac{\pi}{12}$  (B)  $\frac{\pi}{6}$   
 (C)  $\frac{\pi}{4}$  (D)  $\frac{\pi}{3}$

- (E) None of the above.

11. The percentage  $p$  of homes that have purchased a household robot is given by

$$p = \frac{100(2^{t/3})}{16 + 2^{t/3}}$$

where  $t > 0$ ;  $t$  is the time in years since the year 2030. If 50% of the homes have a household robot, what is the date?

- (A) 2011      (B) 2033      (C) 2034      (D) 2042      (E) 2054

12. The number of hours of daylight  $h$  in a day as a function of  $t$ , the number of days since the year started, can be represented by a sine curve of the form

$$h = B + A \sin(\omega t + \theta).$$

For a certain city the longest day has 14 hours of daylight, the shortest day has 10 hours of daylight, and the spring equinox (when the hours of daylight = the hours of darkness) is March 22, which is day  $t = 81$  of the year. Which of the following expressions is the correct representation of the hours of daylight as a function of  $t$  for this city?

(A)  $h = 12 + 2 \sin\left(\frac{2\pi t}{365} - \frac{162\pi}{365}\right)$

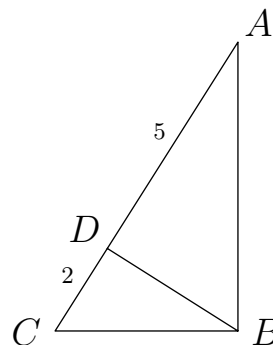
(B)  $h = 10 + 4 \sin\left(\frac{2\pi t}{365} + \frac{162\pi}{365}\right)$

(C)  $h = 10 + 4 \sin\left(\frac{\pi t}{365} + \frac{81\pi}{365}\right)$

(D)  $h = 12 + 2 \sin\left(\frac{\pi t}{365} - \frac{81\pi}{365}\right)$

(E)  $h = 12 + 2 \sin\left(\frac{\pi t}{365} + \frac{81\pi}{365}\right)$

13. In the figure shown, angles  $ABC$  and  $BDC$  are right angles. The length of segment  $AD$  is 5 and the length of segment  $DC$  is 2. Find the length of side  $BC$ .



- (A) 2      (B)  $\sqrt{10}$       (C)  $\sqrt{14}$       (D)  $\sqrt{29}$       (E)  $\sqrt{35}$

14. The hour hand on a clock is 6 cm long. If the tips of the minute hand and the hour hand are 10 cm apart at 3 o'clock, how far apart (in centimeters) are they an hour later?

- (A)  $2\sqrt{13}$  (B)  $2\sqrt{37}$   
 (C)  $10 + 2\sqrt{12}$  (D)  $\sqrt{100 - 48\sqrt{3}}$   
 (E)  $\sqrt{100 + 48\sqrt{3}}$

15. Given that A is the point (12, 5), B is the point (5, 12), and O is the point (0,0), find the cosine of the angle AOB.

- (A) 0 (B)  $\frac{\sqrt{120}}{13}$  (C)  $\frac{\sqrt{3}}{2}$  (D)  $\frac{7}{13}$  (E)  $\frac{120}{169}$

16. A restaurant has a fixed price menu of \$36 for a complete dinner. The average number of customers per night is 200. The owner estimates that for each dollar increase in price, there will be, on average, four fewer customers per night. Estimate the price for a dinner that will produce the maximum amount the owner can expect to receive.

- (A) \$38 (B) \$43  
 (C) \$40 (D) \$45  
 (E) None of the above

17. The negative rational number  $x$  can be written in lowest form as  $x = -p/q$  with  $p$  and  $q$  positive integers and satisfies the equation  $\sqrt{5^x} - 25(5)^{3x} = 0$ . What is  $p - q$ ?

- (A)  $-2$  (B)  $-1$  (C) 0 (D) 1 (E) 2

18. A pyramid has a rectangular base with area 100 square inches, and its other four faces are equilateral triangles. What is the height, in inches, of the pyramid?

- (A)  $5\sqrt{2}$  (B)  $5\sqrt{3}$  (C) 10 (D)  $10\sqrt{2}$  (E)  $10\sqrt{3}$

19. Which of these things is true of the function

$$f(t) = \frac{t}{1-t}?$$

- I. Its domain is the set of all real numbers.  
 II. Its range is the set of all real numbers.  
 III. It is its own inverse.

- (A) All three are true. (B) None are true.  
 (C) Only I is true. (D) Only II is true.  
 (E) Only III is true.

20. Let  $S$  be the set of all numbers which are the sum of the squares of three consecutive integers. Then we can say that
- (A) No member of  $S$  is divisible by 2
  - (B) No member of  $S$  is divisible by 3
  - (C) No member of  $S$  is divisible by 5
  - (D) No member of  $S$  is divisible by 7
  - (E) For any prime number  $N$  there is an element of  $S$  divisible by  $N$
21. Suppose you are in charge of arranging places for six knights around the Round Table. If Sir Lancelot can't sit next to Sir Gawain, Sir Gawain must sit next to Sir Kay, and Sir Kay can't sit across from Sir Tristan (but Sirs Bob and Doug don't care where they sit), how many ways can the six knights be seated? (If two seating arrangements differ only by rotation, you should count them as the same.)
- (A) 14
  - (B) 24
  - (C) 28
  - (D) 36
  - (E) 42
22. In the game *Buzz*, a circle of people take turns counting upwards: "1, 2, 3, 4, . . ." but whenever they get to a number which is either a *multiple* of 7 or which has at least one digit which is a 7, they say "buzz" instead of the number. If a group of people played the game perfectly, starting at 1 and ending at 200, how many times would they say "buzz"?
- (A) 60
  - (B) 61
  - (C) 63
  - (D) 65
  - (E) 68
23. A spherical melon has diameter 8 inches. You cut it in half through the center, and then scoop out the seeds, leaving a 4-inch diameter hemispherical hollow in each half. After you've quartered the melon (that is, cut each half in half again to produce four equal congruent pieces), what is the total surface area of all the pieces?
- (A)  $128\pi$
  - (B)  $132\pi$
  - (C)  $176\pi$
  - (D)  $224\pi$
  - (E)  $(932/3)\pi$
24. Five students are assigned seats in a row of 5 seats. If the students instead sit down at random, find the probability that at least 3 students will sit in their assigned seat.
- (A)  $\frac{3}{5}$
  - (B)  $\frac{1}{2}$
  - (C)  $\frac{1}{10}$
  - (D)  $\frac{1}{6}$
  - (E) None of the above
25. Ned says, "A year from now, I will be 4 times as old as my sister was when my brother was born. A year after that, my brother will be 4 times as old as I was when my sister was born. That year, my sister will be 7 times as old as our dog Frieda." All the ages are positive integers. How old is Ned's brother?
- (A) Not enough information is available.
  - (B) 6
  - (C) 18
  - (D) 30
  - (E) Ned is lying.

## 2005 Answers / Level 2 Test

1. A
2. A
3. D
4. A
5. C
6. C
7. B
8. B
9. D
10. B
11. D
12. A
13. C
14. B
15. E
16. B
17. B
18. A
19. B
20. B
21. C
22. B
23. A
24. E
25. C