College of Charleston
Math Meet 2005
Written Test – Level 3

1. Ship A is steaming north at a steady 10 miles per hour. Ship B, which is 5 miles west of ship A, is steaming east at a steady 15 miles per hour. At what rate is the distance between them changing one hour later?

   (A) -15 mph (B) $25/\sqrt{2}$ mph (C) 15 mph

   (D) $5/3$ mph (E) $-5/3$ mph

2. Find the largest of these 5 numbers.

   (A) $9-3\sqrt{7}-3\sqrt{3}+\sqrt{21}$ (B) $6-2\sqrt{7}-3\sqrt{3}+\sqrt{21}$

   (C) $-4+\sqrt{7}$ (D) $4-\sqrt{7}-4\sqrt{3}+\sqrt{21}$

   (E) 0

3. Claire is a little dizzy from too much sun at the beach and she starts walking in a strange way:

   - she takes one step forward,
   - she turns 90° to her right and then takes two steps forward,
   - she turns 90° to her right and then takes one step forward,
   - she turns 90° to her left and then takes one step backward,
   - she starts all over again.

   Each step is 1 yard. After 186 steps, Claire passes out. How many yards from where she started does Claire end up?

   (A) 186 (B) 1 (C) 2 (D) $\sqrt{2}$ (E) $\sqrt{5}$

4. If p, q, and r are three real numbers, $p\times(q+r)=(p\times q)+(p\times r)$ is always true. In which cases is $p+(q\times r)=(p+q)\times(p+r)$ also true?

   (A) if and only if $p=q=r=\dfrac{1}{3}$ or $p=0$

   (B) if and only if $p=q=r$

   (C) Never

   (D) if and only if $p+q+r=1$ or $p=0$

   (E) if and only if $p=q=r=0$

5. Professor Gollub never lies except on one day of the week (always the same) during which he lies all the time. On how many days of the week can Professor Gollub say *If I did not lie yesterday, I will certainly lie tomorrow*?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. Jeb has to eat a fixed daily quantity of carbohydrates coming from bread or pasta. For example, he can eat 80 grams of pasta and 40 grams of bread, or 100 grams of pasta and 30 grams of bread. If he wanted to eat only pasta, how many grams is he allowed to eat?

   (A) 80 grams (B) 110 grams (C) 140 grams

   (D) 160 grams (E) 200 grams

7. Which of the following is always equal to $\cos^2 A - \sin^2 A$?

(A) $\sin(2A)$

(B) $\cos(A+B)\cos(A-B) - \sin(A+B)\sin(A-B)$

(C) $\sin(A+B)\cos(A-B) - \cos(A+B)\sin(A-B)$

(D) $\cos(A+B)\sin(A-B) - \sin(A+B)\cos(A-B)$

(E) $\cos(2B)$

8. A teacher tried to divide a bag of pennies among her 3 favorite students, but after she gave each of them the same number of pennies, there was one penny left over. She then tried to divide the pennies equally among her 5 favorite students, but there was a penny left over, then among her 7 favorites, but there was a penny left over. Assuming she had more than one penny, what's the fewest number of pennies she could have had in her bag? The first (leftmost) digit in the correct answer is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

9. Let $f(x) = \dfrac{x}{x+1}$. If $f_2(x) = f(f(x))$, and $f_3(x) = f(f(f(x)))$, and $f_4(x) = f(f(f(f(x))))$, etc., what is $f_{99}(x)$?

(A) $\dfrac{x}{99x+1}$ (B) $\dfrac{x}{100x+1}$ (C) $\dfrac{99x}{100x+1}$

(D) $\dfrac{x^{99}}{x^{100}+1}$ (E) $\dfrac{x^{99}}{(x+1)^{99}}$

10. Find the slope of $y = 2|x| - 3|x-2| + 5|3-4x|$ at $x = 1$.

(A) -21 (B) -15 (C) 4 (D) 19 (E) 25

11. Find $\dfrac{dy}{dx}$ where $\ln\, y = \sin(x+y)$

(A) $\dfrac{y\cos(x+y)}{1 - y\cos(x+y)}$ (B) $y\cos\, x$

(C) $\dfrac{\cos(x+y)}{y}$ (D) $\dfrac{1}{y\cos(x+y)}$

(E) $\dfrac{\cos(x+y)}{\sin(x+y)}$

12. At how many different points in the plane does the graph of the polar equation $r = 3\cos(3\theta)$ intersect the unit circle?

(A) 3 (B) 6 (C) 9 (D) 12 (E) 18

13. $(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{63} 64) =$

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

14. Water is pumped into a tank in the shape of an inverted cone at a rate of 5 cubic feet per minute. If the height of the tank is 10 feet and its radius at the top is 4 feet, how fast is the depth of the water increasing when it is 5 feet deep?

(A) $5/(12\pi)$ ft/min (B) $5/(3\pi)$ ft/min (C) 5π ft/min

(D) $5/(4\pi)$ ft/min (E) $5\pi/3$ ft/min

15. Find the intersection of all intervals having the form

$$\left(1-\frac{1}{n},\, 5+\frac{4}{n}\right),$$

where n is a positive integer.

(A) $(0,6)$ (B) $[1,6]$ (C) $[1,5]$ (D) $(1,5)$ (E) $[0,9]$

16. A person in a jeep is launching a person in a glider from an airstrip on a mountain top ridge. The glider is being pulled by a 500 foot chord. One end of the chord is attached to a fixed post at the beginning of the airstrip. The other end is attached to the glider at the same end of the airstrip. The chord goes through a pulley fixed to the back of the jeep. Initially, the jeep is at rest 250 feet down the airstrip. How fast is the glider moving when the jeep is moving forward at 30 mph?

(A) 15 mph (B) 30 mph (C) 60 mph

(D) 90 mph (E) 120 mph

17. A railroad train has forty cars: 10 freight cars, 25 coal cars and 5 passenger cars. All of the freight cars look the same, as do the coal and passenger cars. How many different looking trains can be arranged using these cars?

(A) $40!$

(B) $25!10!5!$

(C) $40!/[(25!)(10!)(5!)]$

(D) $40!/[(40-10)!(40-25)!(40-5)!]$

(E) $40!/[(40-10)!+(40-25)!+(40-5)!]$

18. Find the domain of $f(x)=(1+\sec^{-1}x)(1+\cos^{-1}x)$.

(A) $[0,\pi]$ (B) $\{0,\pi\}$

(C) $[-1,1]$ (D) $(-\infty,-1]\cup[1,\infty)$

(E) $\{-1,1\}$

19. Find the range of $f(x)=\dfrac{\ln x}{x}$.

(A) $(-\infty,e]$ (B) $(-\infty,e)$ (C) $(-\infty,1]$

(D) $(-\infty,\dfrac{1}{e})$ (E) $(-\infty,\dfrac{1}{e}]$

20. I've caught a fish and wish to weigh it. Ordinarily, I would hang the fish by a piece of lightweight line to a scale and read off the weight, but this fish is too heavy for either of my two scales. So, I've hung both scales some distance apart and hung the fish from two fishing lines, one going to each scale. Both lines make a 45-degree angle with the horizontal, and each scale reads 20 lbs. How heavy is my fish?

(A) $20\sqrt{2}$ lbs. (B) $\dfrac{20}{\sqrt{2}}$ lbs. (C) 20 lbs.

(D) 40 lbs. (E) $40\sqrt{2}$ lbs.

21. $(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6})^4 =$

(A) $\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$ (C) $-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$

(D) $-\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$ (E) -1

22. $\sin^{-1}(3/5) + \sin^{-1}(4/5) =$

(A) 0 (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$ (E) π

23. The number of solutions x to the equation

$$|x| + |x-1| + |x-2| + \cdots + |x-100| = m$$

depends on m. For example, there are no solutions if $m = -1$. By choosing m correctly, what's the greatest number of solutions possible?

(A) 1

(B) 2

(C) 3

(D) 4

(E) For the right m, there are infinitely many solutions.

24. When the polynomial $p(x)$ is divided by $x^2 - 1$, the remainder is $x + 2$. When $p(x)$ is divided by $x^2 - 4$, the remainder is $x + 1$. Find the remainder when $p(x)$ is divided by $(x-1)(x-2)$.

(A) $(x+1)(x+2)$ (B) $x+1$

(C) $x-1$ (D) $x+2$

(E) 3

25. An impulse turbine consists of a high speed jet of water striking circularly mounted blades. The power P developed by such a turbine is directly proportional to the speed V of the jet, the speed U of the turbine and the speed of the jet relative to the turbine $V - U$. That is $P = kVU(V-U)$. For a given jet speed V, determine the turbine speed U that will develop maximum power.

(A) V (B) $V/2$ (C) $2V/3$ (D) $3V/4$ (E) $4V/5$

2005 Answers / Level 3 Test

1. B
2. A
3. C
4. D
5. D
6. D
7. B
8. A
9. A
10. E
11. A
12. B
13. D
14. D
15. C
16. C
17. C
18. E
19. E
20. A
21. C
22. D
23. B
24. E
25. B