

**College of Charleston**  
**Math Meet 2007**  
**Written Test – Level 2**

1. There are some marbles in a bag. Amanda says: "There are 3 marbles in the bag and they are all black". Joe says: "There are two black marbles and two red marbles in the bag". Bill says: "There are only black marbles in the bag". If you know that only one person lies, how many marbles are there?  
(A) 1 (B) 2  
(C) 3 (D) 4  
(E) Not enough information
2. The number  $N_2$  is 25% more than the number  $N_1$ , the number  $N_3$  is 20% more than  $N_2$ , and the number  $N_4$  is  $x\%$  less than  $N_3$ . For what value of  $x$  is  $N_4 = N_1$ ?  
(A) 5 (B) 45  
(C)  $33\frac{1}{3}$  (D)  $20\frac{1}{4}$   
(E) None of the above
3. The center of a sphere S is one of the vertices of a cube Q, and the radius of the sphere has the same length as one of the edges of the cube. Then the volume of the intersection of S and Q is:  
(A)  $\frac{1}{8}$  the volume of the sphere (B)  $\frac{1}{4}$  the volume of the sphere  
(C)  $\frac{1}{6}$  the volume of the cube (D)  $\frac{1}{4}$  the volume of the cube  
(E) half the volume of the cube
4. In a certain examination it is noted that the average mark for those passing is 65, while the average of those failing is 35. If the average mark of all participants is 53, what percentage of the participants passed?  
(A) 35 (B) 40 (C)  $66\frac{2}{3}$   
(D) 65 (E) none of these
5. The 5000 students in the GoldenMean group drank a potion that forces them to always tell the truth. While the 5000 students in the ImaginaryPart group drank a potion that forces them to always lie. The 10,000 students form a (large) circle and, facing the center of the circle, each writes on a piece of paper "my right neighbor belongs to the GoldenMean group" or "my right neighbor belongs to the ImaginaryPart group". Obviously, each person knows to which group(s) their neighbors belong. What is the largest possible number of pieces of paper with written on it "my right neighbor belongs to the GoldenMean group"?  
(A) 9,998 (B) 7,500  
(C) 10,000 (D) 5,000  
(E) None of the above

6. The larger rectangle  $ABCD$  has been divided into a square and three rectangles. The area of the square  $AEFG$  is  $x^2$ . The area of rectangle  $FGID$  is  $4x$ . The area of rectangle  $EBHG$  is  $8x$ . Find the area of rectangle  $GHCI$ .
- (A) 16                      (B) 32                      (C) 36  
(D) 48                      (E) none of these
7. If  $a$ ,  $b$ , and  $c$  are three consecutive odd integers such that  $a < b < c$ , find the value of  $a^2 - 2b^2 + c^2$ .
- (A) 0                      (B) 2                      (C) 4                      (D) 6                      (E) 8
8. Which of the following can be lengths of the sides of a triangle?
- (A) 3, 6 and 9                      (B) 4, 5 and 8                      (C) 2, 3 and 8  
(D) 1, 4, 7                      (E) none of these
9. A telephone company offers two plans for long distance service. Plan A cost \$15 a month plus 7 cents per minute of long distance calls. Plan B has no monthly fee but costs 10 cents per minute of long distance calls. When is plan A less expensive than plan B?
- (A) If you talk for more than 8 hours and 20 minutes.  
(B) If you talk for more than 6 hours and 28 minutes.  
(C) If you talk for more than 2 hours and 30 minutes.  
(D) If you talk for less than 6 hours and 28 minutes.  
(E) If you talk for less than 2 hours and 30 minutes.
10. Statistics cited in a recent magazine revealed that the averaged (combined) weekly spending of a teenage boy and girl amounts to \$49.04. Of this total, on average, girls spend \$25.75 per week. Boys spend an average of \$5.90 each week on entertainment. Finally, \$37.89 of the amount spent each week is spent on things other than entertainment. Determine how much money boys spend each week on things other than entertainment.
- (A) \$11.49                      (B) \$17.39                      (C) \$22.64  
(D) \$23.29                      (E) none of these
11. A company works two shifts per day: a morning shift and an evening shift. Their records show that on any given day, two percent of the morning shift employees and one percent of the evening shift employees will miss work. Sixty percent of the workers work the morning shift. What percent of employees are absent on any given day?
- (A) 0.16%                      (B) 1.6%                      (C) 3.0%  
(D) 5.2%                      (E) none of these

12. Let  $y_1 = \frac{x+1}{x-1}$ . Let  $y_2$  be the simplified expression obtained by replacing  $x$  in  $y_1$  by  $\frac{x+1}{x-1}$ . Let  $y_3$  be the simplified expression obtained by replacing  $x$  in  $y_2$  by  $\frac{x+1}{x-1}$ , and so forth. Find  $y_{501}$ .

- (A)  $\frac{x+1}{x-1}$  (B)  $\frac{x-1}{x+1}$   
 (C)  $x$  (D)  $501x$

(E) None of the above

13. Let  $A$  be the set of even integers,  $B$  be the set of integers whose squares are less than 10, and  $C = \{1, 2, 4\}$ . How many elements are in the set

$$(A \cap B) \cup C?$$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

14. Let  $f(n)$  be the square of the sum of the digits of  $n$ . Denote with  $f^{(2)}(n) = f(f(n))$ ,  $f^{(3)}(n) = f(f(f(n)))$  and so on. Compute  $f^{(2007)}(11)$ .

- (A) 1 (B) 256 (C) 16 (D) 169 (E) 251753

15. Every member of the AMS society has the right to vote in the presidential election. The current president was elected with twice as many votes as his only opponent. Knowing that 3 members did not vote, and that the elected president obtained 64% of the votes of the eligible voters, how many members does the AMS society have?

- (A) 69 (B) 81 (C) 75 (D) 87 (E) 99

16. How many real solutions does the following equation have?

$$||a| + 3| - 2| = 1$$

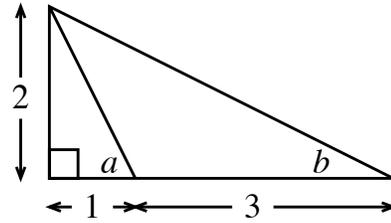
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

17. A student wishing to give 25 cents to each of several charities finds that he is 10 cents short. If, instead, he gives 20 cents to each of the charities, then he is left with 25 cents. Find the amount of money with which the student starts.

- (A) \$7 (B) \$6  
 (C) \$1.65 (D) \$1.60

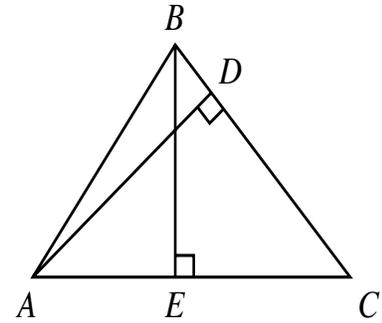
(E) None of the above

18. Find the true statement about the angles  $a$  and  $b$  in the accompanying figure.



- (A)  $75^\circ < a + b \leq 80^\circ$                       (B)  $80^\circ < a + b \leq 85^\circ$   
 (C)  $85^\circ < a + b \leq 90^\circ$                       (D)  $90^\circ < a + b \leq 95^\circ$   
 (E) None of the above.
19. The rows and columns of an 8 by 8 chessboard are numbered 1 through 8. On each square, Bernie puts a number of tokens equal to the sum of the row number and column number for the given square. How many tokens does Bernie put on the chessboard?  
 (A) 482            (B) 576            (C) 768            (D) 1026            (E) 1152
20. The students of SuperMath High School are divided into five groups: GoldenMean, ImaginaryPart, Fibonacci, AbsoluteValue, and LessOrEqual. Each pair of groups are either allies or enemies. Every group which is an ally of the enemy of some other group is also an enemy of that group. How many possible different relationships are there among the five groups?  
 (A)  $2^{10}$     (B) 10  
 (C) 127    (D) 52  
 (E) None of the above
21. If  $b > 0$  and  $f(x) = a + \cos^{-1}(x/b)$ , find  $b - a$  such that the domain and range of  $f$  are equal.  
 (A)  $\pi$             (B)  $-\pi$             (C)  $\pi/2$             (D)  $-\pi/2$             (E) 0
22. On a clock, the hands move continuously so that the minute hand makes a complete revolution every hour and the hour hand makes a complete revolution every twelve hours. Exactly how many hours are there between the times that the hands happen to form a right angle?  
 (A) It is not always the same amount of time between right angle configurations.  
 (B) Always  $\frac{3}{8}$  of an hour.  
 (C) Always  $\frac{4}{9}$  of an hour.  
 (D) Always  $\frac{5}{10}$  of an hour.  
 (E) Always  $\frac{6}{11}$  of an hour.

23. If the measure of  $AC$  is 12, the measure of  $BC$  is 10, and the measure of  $AD$  is 9, then the measure of  $BE$  is

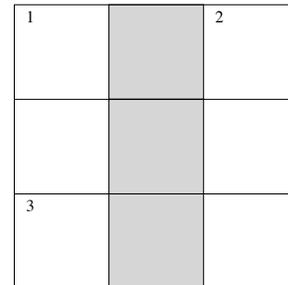


- (A) between 6 and 7  
 (B) between 7 and 8  
 (C) between 8 and 9  
 (D) between 9 and 10  
 (E) none of these
24. If  $a$  is a non-zero real number, and if  $p(x)$  and  $q(x)$  are real, non-constant polynomials such that

$$p(x) \cdot q(x) = x^4 + a^4,$$

what is the product of the coefficients of their linear terms?

- (A)  $2a^2$   
 (B)  $-2a^2$   
 (C)  $-4a^2$   
 (D) 0  
 (E) It cannot be determined from the given information.
25. Here is a crossword for you, but each entry is a digit instead of a letter. Each digit between 1 and 9 appears in the crossword once and only once. You are given the following clues:



**Across:**

1. A square.  
 3. A number the sum of whose digits is 9.

**Down:**

1. A cube.  
 2. A number with only even digits.

What is the number in the gray column, read from top to bottom?

- (A) 820  
 (B) 937  
 (C) 764  
 (D) 793

- (E) None of the above

## 2007 Answers / Level 2 Test

1. C
2. C
3. A
4. E
5. A
6. B
7. E
8. B
9. A
10. B
11. B
12. A
13. D
14. B
15. C
16. B
17. C
18. C
19. B
20. D
21. A
22. E
23. B
24. B
25. D