College of Charleston
Math Meet 2008
Written Test – Level 3

1. From a gray square of sidelength 1 (Fig. 1), remove a square of sidelength 1/3 (Fig. 2), then eight squares of sidelength 1/9 (Fig. 3), etc. (Fig.s 4–6). If this process is repeated ad infinitum, what will be the remaining gray area?

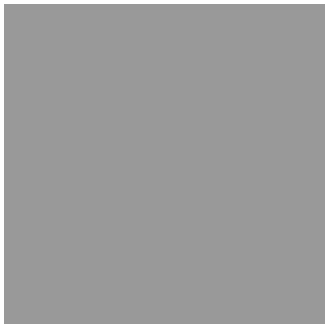


Fig. 1

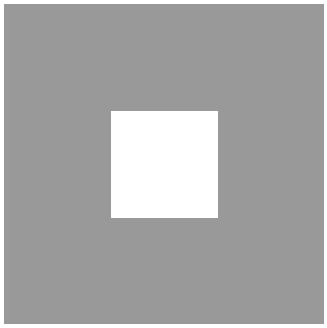


Fig. 2

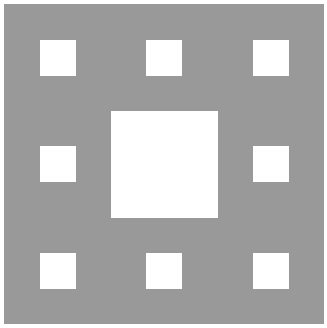


Fig. 3

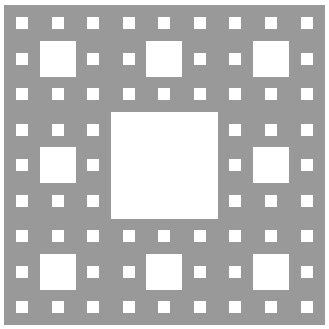


Fig. 4

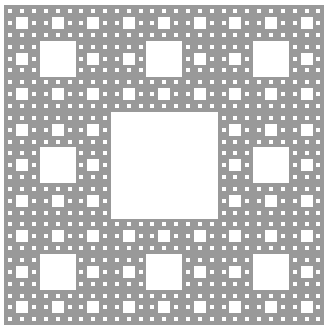


Fig. 5

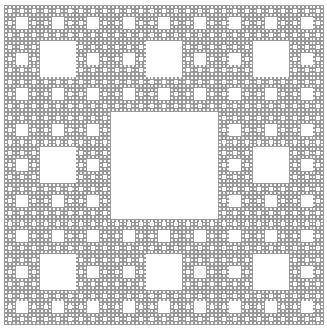


Fig. 6

(A) 0 (B) 1/9 (C) 2/243 (D) 1/243 (E) 1/729

2. Use the information that $g(2) = 3$, $g(4) = 5$, $g'(2) = -4$, and $g'(4) = -6$ to find $f'(2)$ for $f(x) = x^2 \ln(g(x))$.

(A) $-\frac{16}{6} + 8\ln(3)$ (B) $-\frac{16}{3} + 4\ln(3)$

(C) $4\ln(3)$ (D) $-\frac{16}{3}$

(E) $-\frac{16}{3} + 4\ln(6)$

3. For every positive integer n, let S_n denote the number of sequences of length n of 0s and/or 1s that do not contain two consecutive 1s. Suppose S_n is odd and S_{n+2} is even. Then

(A) n must be divisible by 3. (B) $n+1$ must be divisible by 3.

(C) n must be divisible by 4. (D) $n+1$ must be divisible by 4.

(E) $n+2$ must be divisible by 4.

4. The graph of the equation below consists of three lines. Find the sum of their slopes.

$$y^3 - 21x^2y - 20x^3 + 5y^2 - 17xy - 94x^2 + 2y - 58x - 8 = 0$$

(A) 0 (B) 5 (C) 16 (D) 25 (E) 94

5. Find the area of the parallelogram with vertices $(1, 2)$, $(2, 5)$ $(-2, 7)$, and $(-3, 4)$.

(A) 7 (B) 10 (C) 14 (D) 24 (E) 28

6. Find the product: $\left(\cos\frac{x}{2}\right)\left(\cos\frac{x}{4}\right)\left(\cos\frac{x}{8}\right)\cdots\left(\cos\frac{x}{256}\right)$

(A) $\dfrac{\sin x}{128\sin\frac{x}{256}}$ (B) $\dfrac{\sin x}{256\sin\frac{x}{512}}$ (C) $\dfrac{\sin x}{128\sin\frac{x}{128}}$

(D) $\dfrac{\sin x}{256\sin\frac{x}{256}}$ (E) $\dfrac{\sin x}{512\sin\frac{x}{512}}$

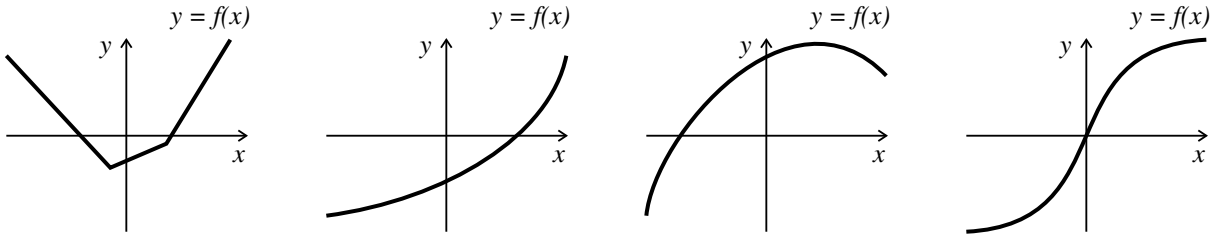
7. Find the exact positive value of c that makes the area under the graph of $y = c(1 - x^2)$ and above the x-axis equal to 1.

(A) 3/4 (B) 4/3

(C) 4 (D) 3

(E) cannot be determined

8. How many of the functions graphed satisfy $\dfrac{f(a) + f(b)}{2} \geq f\left(\dfrac{a+b}{2}\right)$ for all real numbers a and b?



(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. Five Hatfields and eight McCoys are to sit at a round table. After Pappy McCoy takes his seat, how many ways can the other 12 people be seated so that no Hatfield is next to another Hatfield?

(A) $\frac{7!5!}{2!}$ (B) $\frac{7!5!}{3!}$ (C) $\frac{5!8!}{3!}$ (D) $\frac{7!8!}{3!}$ (E) $\frac{8!8!}{3!}$

10. Find the number of positive integers $n \leq 100$ for which $\frac{n(n+1)(n+2)}{12}$ is an integer.

(A) 8 (B) 25 (C) 50 (D) 75 (E) 80

11. In a Sudoku puzzle, one must place a digit (1,2,. . . ,9) in each of the 1×1 squares so that each row, each column, and each of the 3×3 grids outlined in bold contains all 9 digits. Find the digit that must go in the shaded square.

			7		9			
	3	2				5	1	
4								9
		5	3		1	8		
	1						9	
		7	9		8	2		
7								2
	5	4				1	8	
			5		4			

(A) 2 (B) 3 (C) 6 (D) 7 (E) 9

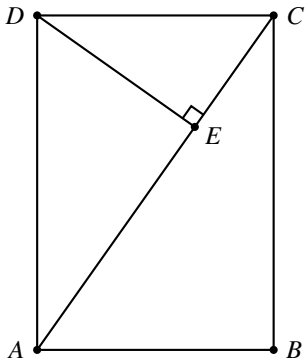
12. Find the sum of all three-digit positive integers which are multiples of 9.

(A) 54,351 (B) 55,449 (C) 54,540 (D) 55,350 (E) 56,349

13. In the binomial $(x-y)^{16}$, the coefficient of x^4y^{11} is

(A) 0 (B) 420 (C) 560 (D) 1820 (E) -3360

14. Find the area of the rectangle $ABCD$ if (the length) $\overline{EC} = 8$, and $\overline{AE} = \overline{ED} \times \sqrt{2}$.
(Figure not to scale.)



(A) $192\sqrt{2}$ (B) $196\sqrt{2}$ (C) $198\sqrt{2}$
(D) $188\sqrt{2}$ (E) $164\sqrt{2}$

15. How many distinct real solutions are there to the equation

$$x^{199} + 2x^{198} + 4x^{197} + \cdots + 2^{197}x^2 + 2^{198}x + 2^{199} = 0?$$

(A) 200 (B) 199 (C) 2 (D) 1 (E) 0

16. Find the sum of the following infinite collection of numbers:

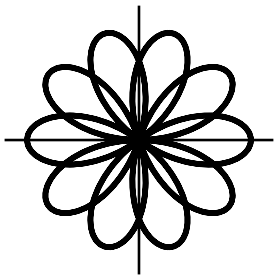
$$\begin{array}{ccccccc} 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \frac{1}{81} & \cdots \\ & \frac{1}{6} & \frac{1}{18} & \frac{1}{54} & \frac{1}{162} & \cdots \\ & & \frac{1}{36} & \frac{1}{108} & \frac{1}{324} & \cdots \\ & & & \frac{1}{216} & \frac{1}{648} & \cdots \\ & & & & \frac{1}{1296} & \cdots \\ & & & & & \ddots \end{array}$$

(A) $\frac{5}{3}$ (B) $\frac{11}{7}$ (C) $\frac{9}{5}$ (D) $\frac{35}{18}$ (E) $\frac{7}{4}$

17. If $f(x) = Ax^3 + Bx^2 + Cx + D$ and if the graph of $y = f(x)$ passes through the points $(-1, 1)$, $(0, 4)$, and $(1, 2)$, evaluate $\int_{-1}^{1} f(x)\,dx$.

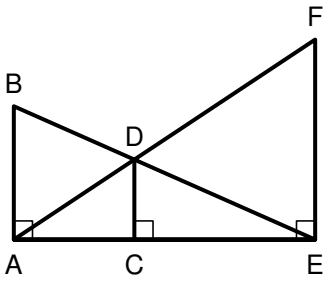
(A) $13/3$
(B) $15/3$
(C) $17/3$
(D) $19/3$
(E) Not enough information is given to determine the integral.

18. Which of the polar equations is graphed in the accompanying figure?



(A) $r = \cos 1.5\theta$ (B) $r = \cos 2.5\theta$ (C) $r = \cos 3.5\theta$
(D) $r = \cos 4.5\theta$ (E) $r = \cos 5.5\theta$

19. If the distance AB is 10 and EF is 15, find the distance CD.



(A) 9
(B) 8
(C) 7.5
(D) 6
(E) Not enough information to determine CD

20. Every function $f(x)$ is the sum of an even function $f_e(x)$ and an odd function $f_o(x)$. Find $f_e(x)$.

(A) $(f(x))^2 - (f(-x))^2$ (B) $f(|x|)$
(C) $\dfrac{f(x) - f(-x)}{2}$ (D) $|f(x)|$
(E) $\dfrac{f(x) + f(-x)}{2}$

21. A is the set $\{1, 2, 3, 4\}$. B and C are nonempty sets. $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B \cap C = \emptyset$. If the number of elements of B is greater than the number of elements in C, find the smallest possible sum of the elements of B.

(A) 6 (B) 8 (C) 10 (D) 11 (E) 13

22. If $-1 < x < y < 0 < u < 1 < v$, which of the following could be true?

(A) $yu = x$ (B) $x = y/v$
(C) $\ln x + \ln y = \ln u.$ (D) $u + \ln v = v$
(E) $\ln |x| = x$

23. Two planets travel at constant speeds in circular orbits centered about their sun. Both travel in the same direction about the sun, say, clockwise. One orbits the sun once every year, while the other orbits the sun once every 2.25 years. If the sun and two planets are in a straight line now, how many years must pass before this is true again? In the correct answer, the first digit to the right of the decimal is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

24. You are told that a paper bag is filled with two kinds of candies: lemon drops and mint drops. Each candy is wrapped in either red or white paper. You are told that when you randomly select a candy from the bag, the probability is 0.8 that it will have a red wrapper. Furthermore, you learn that the probability that a randomly selected candy from the bag is a mint drop with a red wrapper is 0.2.
You reach in the bag and pull out a candy with a red wrapper. What is the probability that it is a mint drop?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) 0

25. Let $i = \sqrt{-1}$. Then $(1+i)^{17} - (1-i)^{17}$ =

(A) 128 (B) $-128i$ (C) $256i$ (D) -256 (E) $512i$

2008 Answers / Level 3 Test

1. A
2. B
3. B
4. A
5. C
6. D
7. A
8. C
9. D
10. D
11. E
12. D
13. A
14. A
15. D
16. C
17. D
18. B
19. D
20. E
21. B
22. E
23. E
24. C
25. E