

**College of Charleston**  
**Math Meet 2009**  
**Written Test – Level 1**

1. The Great Venus Marathon Swim was held last November. It was 2008 miles long, and 80% of the planet's inhabitants participated. After two miles, 95% of participants dropped out. The remaining 2000 swimmers completed the Marathon Swim. How many inhabitants are there on Venus?  
(A) 20,000                      (B) 40,000                      (C) 50,000  
(D) 80,000                      (E) 100,000
2. In how many 4-digit integers does the digit 5 appear once and only once and as the largest digit present?  
(A) 225                      (B) 400                      (C) 425                      (D) 525                      (E) 600
3. Let  $l_1$  be the line  $y = x + 2$ . Let  $l_2$  be a parallel line which passes through the point  $(0, 4)$ . What is the shortest distance from  $l_1$  to  $l_2$ ?  
(A)  $\sqrt{2}$                       (B) 2                      (C)  $\frac{1}{2}$   
(D) 4                      (E) none of these
4. If  $a$ ,  $b$ , and  $c$  are three consecutive odd integers such that  $a < b < c$ , find the value of  $a^2 - 2b^2 + c^2$ .  
(A) 6                      (B) 7                      (C) 8  
(D) 9                      (E) none of these
5. In a given right triangle, the perimeter is 30 and the sum of the squares of the three sides is 338. Find the difference between the length of the hypotenuse and the shorter leg of the triangle.  
(A) 5                      (B) 7                      (C) 8  
(D) 9                      (E) none of these
6. A television producer has a list of ten possible cast members for her new "reality TV" show about three people living together in an apartment in Charleston. So, she has to choose which three people out of those ten will be on the show. How many possibilities does she have to choose between?  
(A) 30                      (B) 570                      (C) 120                      (D) 200                      (E) 9
7. What is the difference between the area of a quadrant of a circle with radius 1 inch and the area of a half circle with diameter 1 inch?  
(A)  $\pi/8 \text{ in}^2$                       (B)  $\pi/4 \text{ in}^2$                       (C)  $\pi/2 \text{ in}^2$   
(D)  $\pi \text{ in}^2$                       (E) none of these
8. In a rectangle ABCD, E is a point on the side CD. The area of the triangle ADE is one fifth of the area of the quadrilateral ABCE. Compute the ratio between the length of DC and the length of DE.  
(A) 2:1                      (B) 3:1                      (C) 4:1                      (D) 5:1                      (E) 6:1

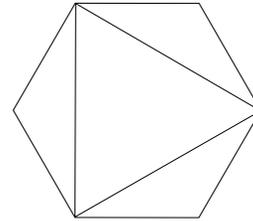
9. A sphere is placed on a horizontal plane during a sunny day. At a certain instant of time, its shadow reaches 10 meters from the point where the sphere touches the plane. At the same time, a 1 meter tall post casts a 2 meter long shadow. What is the radius of the sphere, expressed in meters?

(A)  $\frac{5}{2}$  (B)  $9 - 4\sqrt{5}$  (C)  $10\sqrt{5} - 20$   
(D)  $8\sqrt{10} - 23$  (E)  $6 - \sqrt{15}$

10. How many 3 digit numbers are perfect squares whose final (rightmost) digit is also a perfect square?

(A) 10 (B) 16 (C) 18  
(D) 21 (E) none of these

11. A regular hexagon has side length 1. Using alternating vertices of the hexagon, we form an equilateral triangle. What is the area of the triangle?



(A)  $\frac{1}{2}$  (B) 6 (C)  $\pi$   
(D)  $\frac{3\sqrt{3}}{4}$  (E) none of these

12. Among the primes less than 100, how many are the sum of three consecutive integers?

(A) none (B) 1 (C) 3  
(D) 12 (E) all of them

13. How many values of  $x$  have the property that the median of the numbers  $x, 3, 6, 9, 10$  (not necessarily in increasing order) is equal to the mean of these five numbers?

(A) 1 (B) 2 (C) 3  
(D) 5 (E) none of these

14. How many zero digits are in the number  $(4^{12})(5^{20})$ ?

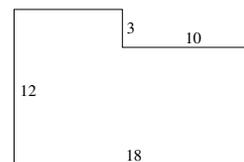
(A) 12 (B) 16 (C) 20  
(D) 22 (E) none of these

15. Side  $AD$  of square  $ABCD$  is a diagonal of square  $AEDF$ . Point  $G$  is the midpoint of segment  $AD$ , and point  $E$  lies in the interior of square  $ABCD$ . If the area of triangle  $AGF$  is equal to  $1/4$ , then what is the area of polygon  $DEABC$ ?

(A) 1.25 (B) 1.5 (C) 1.75  
(D) 2.5 (E) none of these

16. If the remainder is 1 when  $n^2$  is divided by 4, what is the remainder when  $(n + 5)^2$  is divided by 4?  
(A) 0 (B) 1 (C) 2  
(D) 3 (E) none of these
17. Let the set of all real numbers  $x$  satisfying the inequalities  $|x - 8| \leq 6$  and  $|x - 3| \geq 5$  be written as  $[a, b]$ . Find the value of  $b - a$ .  
(A) 6 (B) 8 (C) 12  
(D) 14 (E) none of these
18. A 9 inch piece of string has some marks on it. It is possible to cut the string at some of those marks to end up with a piece of any integer length from 1 to 8 inches. What is the smallest number of marks that could be on the string?  
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7
19. Consider the set  $S = \{1, 2, \dots, 2004\}$ . How many subsets of  $S$  are there such that the sum of their elements equals 2,009,000?  
(A) 8 (B) 10 (C) 16 (D) 1002 (E)  $2^{2000}$
20. There are 20 ping pong balls in a bag, each ball is stamped with an integer between 0 and 10 (0 and 10 allowed). The number stamped on each ball, if not 0, is the sum of the numbers stamped on all the other balls. Then the number of balls stamped with 0 is:  
(A) no more than 5 (B) 10 (C) 13  
(D) 16 (E) at least 18
21. For the polynomial  $x^4 + 3x^3 - 5x^2 + 3x + 1$ , if  $a$  is a root, which of the following must also be a root?  
(A)  $a^2$  (B)  $-a^2$  (C)  $\frac{1}{a}$   
(D)  $-\frac{1}{a}$  (E) none of these
22. Given the polynomial  $R(x) = ax^3 + 2a^2x^2 + a^3$ , find the sum of all real numbers  $a$  for which  $x - 2$  is a factor of  $R(x)$ .  
(A) -16 (B) -8 (C) 8  
(D) 16 (E) none of these
23. A farmer keeps three kinds of animals: cows, chickens and pigs. Last year all but two of his animals were cows, all but two were chickens and all but two were pigs. How many different possibilities are there for the *total* number of animals he had on his farm last year?  
(A) There is only one possibility.  
(B) There are three different possibilities.  
(C) There are infinitely many possibilities.  
(D) None, there is no way this description could be accurate.

24. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that can be drawn on the floor of this room?



- (A)  $16\pi$       (B)  $\frac{81}{4}\pi$       (C)  $25\pi$       (D)  $\frac{145}{4}\pi$       (E)  $841\pi$
25. Let  $S$  be the set of all integers that can be written as the difference of the squares of two integers. What is true about the set  $S$ ?
- I. It contains all even numbers.
  - II. It contains all odd numbers.
  - III. It contains all multiples of three.
  - IV. It contains all multiples of four.
- (A) Only I is true.  
(B) Only II is true.  
(C) Only I and III are true.  
(D) Only II and IV are true.  
(E) None of these statements is true.

## 2009 Answers / Level 1 Test

1. C
2. C
3. A
4. C
5. C
6. C
7. A
8. B
9. C
10. B
11. D
12. B
13. C
14. C
15. B
16. A
17. A
18. A
19. B
20. E
21. C
22. B
23. A
24. C
25. D