College of Charleston
Math Meet 2009
Written Test – Level 2

1. A small commuter plane can carry 8 passengers. The probability that a ticket holder will not show up for the flight is .1, independent of the other ticket holders. If the airline sells 10 tickets for the flight, find the probability that more passengers will show up than there are seats available for the flight.

   (A) $19\left(\frac{9^9}{10^{10}}\right)$ (B) $\frac{1}{2}\left(\frac{9}{10}\right)^9$ (C) $\frac{1}{2}\left(\frac{9}{10}\right)^{10}$

   (D) $\frac{1}{19}\left(\frac{9^{10}}{10^9}\right)$ (E) $\left(\frac{9}{10}\right)^{10}$

2. Let $$f(x) = \alpha x(x-1)(x-2) + \beta x(x-1) + \gamma x + \delta.$$

   Find α if $$f(0) = f(1) = f(2) - 1 = f(3) - 1.$$

   (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{3}$ (E) $-\frac{1}{6}$

3. Among the primes less than 100, how many are the sum of three consecutive integers?

   (A) none (B) 1 (C) 3

   (D) 12 (E) all of them

4. How many solutions to $\sin 2x = \sin x$ are in the interval $[0, 7\pi]$?

   (A) 7 (B) 14 (C) 15 (D) 16 (E) 17

5. How many triples $\{x, y, z\}$ are there such that y is an integer,

   $$x + 1 = y = z - 1 \qquad \text{and} \qquad xyz = x + y + z?$$

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. A **code** consists of a sequence of 10 digits with repetitions allowed, for example, 0034768927. Determine the probability that the sequence **01234** will occur consecutively in a randomly generated code, for example, as it does in 847**01234**62 but does not in 82**0123**6**4**99.

   (A) 5.00001×10^{-5} (B) 5.0×10^{-5}

   (C) 6.0×10^{-5} (D) 6.0×10^{-10}

   (E) 5.99999×10^{-5}

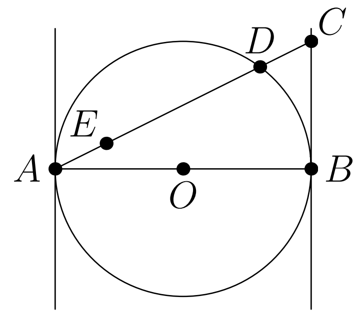
7. Let f be the function defined by

   $$f(n) = 2f(n-1) + 3f(n-2)$$

   where $f(1) = 1$ and $f(2) = 2$. Compute $f(5)$.

   (A) 5 (B) 7 (C) 20 (D) 56 (E) 61

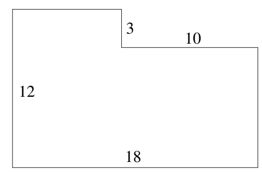
8. In the figure, AB is a diameter of the circle with center O and radius r. A chord AD is drawn and extended until it meets the tangent to the circle at B in point C. Then, point E is taken on AC so that $AE = DC$. Let x be the minimum distance from E to the tangent though A and y be the minimum distance from E to the diameter AB. What must be true about x and y?



(A) $y^2 = \dfrac{x^3}{2r - x}$ (B) $y^2 = \dfrac{x^3}{2r + x}$ (C) $y^4 = \dfrac{x^2}{2r - x}$

(D) $x^2 = \dfrac{y^2}{2r - x}$ (E) $x^2 = \dfrac{y^2}{2r + x}$

9. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that can be drawn on the floor of this room?



(A) 16π (B) $\dfrac{81}{4}\pi$ (C) 25π (D) $\dfrac{145}{4}\pi$ (E) 841π

10. Suppose k is a fixed positive number in the interval $(0, 1)$. What is the sum of all numbers θ in the interval $(0, 2\pi)$ such that $\cos(\theta) = k$?

(A) $\dfrac{\pi}{2}$

(B) π

(C) $\dfrac{3\pi}{2}$

(D) 2π

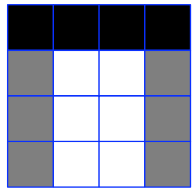
(E) Can be different values depending on the value of k.

11. An integer between 1 and 1,000,000, inclusive, is randomly chosen and found to be a perfect square. What is the probability that it is also a perfect cube?

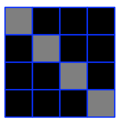
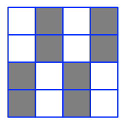
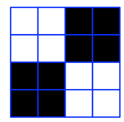
(A) 0 (B) 0.01 (C) 0.02

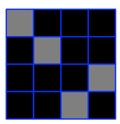
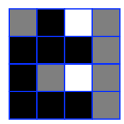
(D) $0.033333\ldots$ (E) 0.045

12. Suppose we have a 4×4 grid of squares and each square can either be white, grey or black. For any given square, there are seven squares that are in the same row and/or column as that square (including the square itself). Call these seven squares "the neighborhood" of that square. Suppose also that when you touch any square, then every white square in its neighborhood becomes grey, any grey square in its neighborhood becomes black, and any black square in its neighborhood becomes white. So, for instance, if we begin with a grid which is all white and touch the top left corner and then touch the top right corner, the result will look like the figure:



The question is, which of the following can you *not* get by starting with a completely white grid and touching *exactly* four squares?

(A)  (B)  (C) 

(D)  (E) 

13. If $2\log(x-2y) = \log x + \log y$, find x/y. Assume that x, y, and $x-2y$ are strictly greater than zero.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. Which of these is equal to $A + 2B + 4C$ if

$$Ax^2 + By^2 - 2x + 3y + C = 0$$

is a circle that passes through the origin?

(A) $A+B+C$ (B) $A+B-C$ (C) $3A$

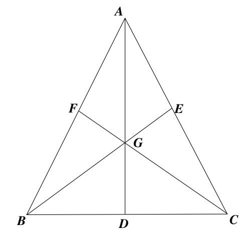
(D) $4B$ (E) ABC

15. A sphere is placed on a horizontal plane during a sunny day. At a certain instant of time, its shadow reaches 10 meters from the point where the sphere touches the plane. At the same time, a 1 meter tall post casts a 2 meter long shadow. What is the radius of the sphere, expressed in meters?

(A) $\frac{5}{2}$ (B) $9-4\sqrt{5}$ (C) $10\sqrt{5}-20$

(D) $8\sqrt{10}-23$ (E) $6-\sqrt{15}$

16. Let ABC be a triangle, and let D, E, F be the midpoints of the sides. Assume that $AB = AC$, and let BE and CF intersect at point G (see diagram). If $BD = x$ and $AG = y$, what is the length AF as a function of x and y?



(A) $\sqrt{x^2 + 2y^2}$ (B) $\frac{1}{2}\sqrt{x^2 + 4y^2}$

(C) $\frac{1}{2}\sqrt{x^2 + 9y^2}$ (D) $\frac{1}{4}\sqrt{x^2 + 9y^2}$

(E) $\frac{1}{4}\sqrt{4x^2 + 9y^2}$

17. For how many integers n between 1 and 100 does $x^2 + x - n$ factor into the product of two linear factors with integer coefficients?

(A) 0 (B) 1 (C) 2 (D) 9 (E) 10

18. Six balls numbered 1 through 6 are randomly dropped into six boxes numbered 1 through 6. That is, one ball is dropped in each box, with an equal probability that any given ball ends up in any given box. We say that a ball is in "the right box" if it has the same number as the box that it is in. Which of these events has the *lowest* probability?

(A) no ball ends up in the right box

(B) exactly one ball ends up in the right box

(C) exactly three balls end up in the right boxes

(D) exactly five balls end up in the right boxes

(E) all of the balls end up in the right boxes

19. Let C be the circle of radius $\sqrt{5}$ centered at $(5, 0)$. Find the set of all numbers m such that $y = mx$ intersects C **twice**.

(A) $[0, 1)$ (B) $[-1, 1]$

(C) $(-1/2, 1/2)$ (D) $(-1/\sqrt{3}, 1/\sqrt{3})$

(E) $[-1/\sqrt{5}, 1/\sqrt{5}]$

20. Which of these describes the entire solution set to the inequality

$$\frac{x^2-4}{1-x^2} > 0?$$

(A) $1 < x < 2$

(B) $1 < |x| < 2$

(C) $x < -2, -1 < x < 1, 2 < x$

(D) $|x| > 2$

(E) $-2 \leq x \leq -1, 1 \leq x \leq 2$

21. A man is driving a car from his home to his office. He travels half of the distance at an average speed of 20 miles per hour. What must his average speed be for the second half of the trip so that his average speed for the total trip will be exactly 40 miles per hour?

(A) The answer is between 40 and 80 mph.

(B) The answer is between 80 and 100 mph.

(C) The answer is between 100 and 120 mph.

(D) We can only answer if we know the total distance traveled.

(E) It is not possible to make the average speed 40 mph for the entire trip.

22. A charter bus company charges $\$10$ per person for a round trip to a ball game with a discount given for group fares. A group purchasing more than 10 tickets at one time receives a reduction per ticket of $\$0.25$ times the number of tickets in excess of 10. What is the maximum revenue per group that can be received by the bus company?

(A) $\$15$ (B) $\$150$

(C) $\$156.25$ (D) $\$126.50$

(E) None of the above

23. What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1}$ is smaller than 0.01?

(A) 101 (B) 1001

(C) 1501 (D) 2501

(E) None of the above

24. What must be true of any six digit number, n, having decimal representation $n = abcabc$?

(A) the numbers a, b and c are factors of n

(B) n satisfies $(n+1)(n-1) = n^2$

(C) n is a multiple of 27

(D) n is a perfect square

(E) 7, 11 and 13 are prime factors of n

25. The equation $a\cos(x+b) = A\sin(x) + B\cos(x)$ is true for all x whenever:

(A) $A = -a\sin b$ and $B = a\cos b$

(B) $A = a\sin b$ and $B = -a\cos b$

(C) $A = a\cos b$ and $B = -a\sin b$

(D) $A = -a\cos b$ and $B = a\sin b$

(E) $A = a\cos b$ and $B = b\sin a$

2009 Answers / Level 2 Test

1. A
2. D
3. B
4. C
5. D
6. E
7. E
8. A
9. C
10. D
11. B
12. B
13. D
14. C
15. C
16. E
17. D
18. D
19. C
20. B
21. E
22. C
23. D
24. E
25. A