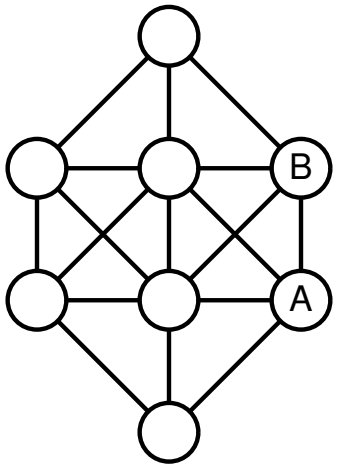
College of Charleston
Math Meet 2009
Written Test – Level 3

1. Find the value of a so that $f(x) = xe^{ax}$ has a critical point at $x = 3$.

   (A) -1/4 (B) 2 (C) -1/3 (D) -1 (E) 2/3

2. Each of the eight circles in the figure is to be filled with exactly one of the digits $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Each digit in this list is to be used exactly once. No two adjacent numbers in this list (2 and 3, for example) may go in circles that are connected by a line. For instance, if A is 2, then B cannot be 1 or 3. (The numbers 1 and 8 are not considered adjacent.)
   Find B if A is 3.



   (A) 1 (B) 5 (C) 6 (D) 7 (E) 8

3. A is the set $\{1, 2, 3, 4\}$. B and C are nonempty sets. $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B \cap C = \emptyset$. If the number of elements of B is less than the number of elements in C, find the greatest possible sum of the elements of B.

   (A) 7 (B) 13 (C) 18 (D) 22 (E) 25

4. Mr. Jones (who commutes to work in the city by train) unexpectedly caught an earlier train than usual yesterday. Normally, his wife drives to the station, meets Mr. Jones, and they drive home together. Not having heard from her husband, Mrs. Jones left home for the station at her usual time yesterday. When he arrived at the station, Mr. Jones set out on foot toward home, met his wife on the way, and they arrived home 12 minutes earlier than they would have if he had waited at the station for his wife's arrival. Due to heavy traffic at rush hour, the car is only able to maintain an average speed that is 5 times the rate at which Mr. Jones is able to walk.
   Mr. and Mrs. Jones arrived home at exactly six o'clock. At what time would he have reached home if he had had time to call his wife before boarding the train so that she could have met him at the station as his train arrived? The correct answer is closest to

   (A) 5:18 (B) 5:24 (C) 5:30 (D) 5:36 (E) 5:42

5. At how many points along the curve $x^2y^2 + xy = 2$ does the tangent line have slope -1?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. Let $$f(x) = \alpha x(x-1)(x-2) + \beta x(x-1) + \gamma x + \delta.$$

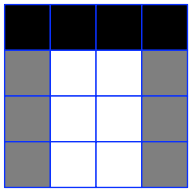
   Find α if $$f(0) = f(1) = f(2) - 1 = f(3) - 1.$$

   (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{3}$ (E) $-\frac{1}{6}$

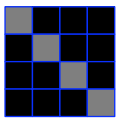
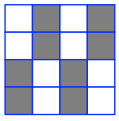
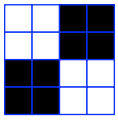
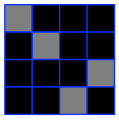
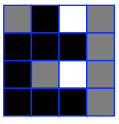
7. Suppose that f is an even, periodic function with period 2, and that $f(x) = x$ for all x in the interval $[0, 1]$. Find $f(3.14)$.

   (A) 3.14 (B) -3.14 (C) $-.14$ (D) $.14$ (E) $.86$

8. Suppose we have a 4×4 grid of squares and each square can either be white, grey or black. For any given square, there are seven squares that are in the same row and/or column as that square (including the square itself). Call these seven squares "the neighborhood" of that square. Suppose also that when you touch any square, then every white square in its neighborhood becomes grey, any grey square in its neighborhood becomes black, and any black square in its neighborhood becomes white. So, for instance, if we begin with a grid which is all white and touch the top left corner and then touch the top right corner, the result will look like the figure:



   The question is, which of the following can you *not* get by starting with a completely white grid and touching *exactly* four squares?

   (A) 
   (B) 
   (C) 
   (D) 
   (E) 

9. If x is very large, then $$\log_3(6x - 5) - \log_3(2x + 1)$$ will be very close to the number

   (A) 0 (B) 1 (C) 3 (D) 4 (E) none of these

10. If x and y are real numbers such that $x + y = 1$ and $xy = -1$, which of the following statements is true?

(A) $x^2 + y^2 = 2$ (B) $x^3 + y^3 = 3$ (C) $x^4 + y^4 = 6.$

(D) $x^5 + y^5 = 11$ (E) $x^6 + y^6 = 17$

11. $\left(\sin\dfrac{3\pi}{10} + i\cos\dfrac{3\pi}{10}\right)^{10}$ equals

(A) 1 (B) -1 (C) i (D) $-i$ (E) $\dfrac{1+i}{\sqrt{2}}$

12. If b is a real number, and if the average rate of change of the function $f(x) = b^x$ on the interval $[2, 4]$ is 3, what is its average rate of change on $[4, 6]$?

(A) 3 (B) 4 (C) 4.5 (D) 6 (E) 9

13. In a given forest, suppose there are more trees than there are leaves on any *single* tree. Which of the following must be true?

I. There exist two trees with a leaf in common.
II. There exist at least two trees which have the same number of leaves.
III. There exists a tree with no leaves.

(A) I only (B) II only (C) III only

(D) either II or III (E) either I or III

14. A staircase has twelve steps. You can take one or two steps at a time. In how many different ways can you go up the staircase?

(A) 116 (B) 228 (C) 233

(D) 427 (E) none of these

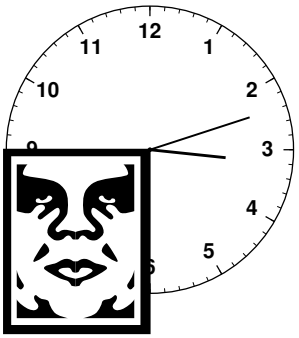
15. If the graph of the polar equation $r = 1 + 3\sin\theta$ is rotated $\pi/2$ radians counterclockwise, find an equation for the resulting graph.

(A) $r = 1 + 3\sin(\theta + \pi/2)$ (B) $r = 1 + 3\cos(\pi/2 - \theta)$

(C) $r = 1 - 3\sin(\theta - \pi/2)$ (D) $r = 1 - 3\cos\theta$

(E) $r = 1 + 3\cos\theta$

16. A sticker covers one quarter of the face of a clock. For what fraction of the day is it not possible to see both hands?



(A) $\dfrac{3}{16}$ (B) $\dfrac{4}{16}$ (C) $\dfrac{5}{16}$ (D) $\dfrac{7}{16}$ (E) $\dfrac{9}{16}$

17. Find the largest of these 5 numbers.

(A) $9 - 3\sqrt{7} - 3\sqrt{3} + \sqrt{21}$ (B) $6 - 2\sqrt{7} - 3\sqrt{3} + \sqrt{21}$

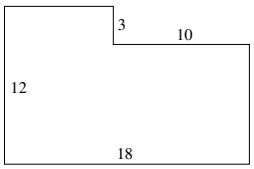
(C) $-4 + \sqrt{7}$ (D) $4 - \sqrt{7} - 4\sqrt{3} + \sqrt{21}$

(E) 0

18. At how many different points in the plane does the graph of the polar equation $r = 3\cos(3\theta)$ intersect the unit circle?

(A) 3 (B) 6 (C) 9 (D) 12 (E) 18

19. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that can be drawn on the floor of this room?



(A) 16π (B) $\frac{81}{4}\pi$ (C) 25π (D) $\frac{145}{4}\pi$ (E) 841π

20. Which of the following is always equal to $\cos^2 A - \sin^2 A$?

(A) $\sin(2A)$

(B) $\cos(A+B)\cos(A-B) - \sin(A+B)\sin(A-B)$

(C) $\sin(A+B)\cos(A-B) - \cos(A+B)\sin(A-B)$

(D) $\cos(A+B)\sin(A-B) - \sin(A+B)\cos(A-B)$

(E) $\cos(2B)$

21. The sum of those positive integers between 10 and 100 which have remainder 1 when divided by 4 is

(A) odd. (B) divisible by 3. (C) divisible by 4.

(D) divisible by 7. (E) divisible by 10.

22. Where does the tangent line to the graph of the function

$$f(x) = x^2 - 3x + 1$$

at the point $(0, 1)$ hit the x-axis?

(A) at $x = -1$ (B) at $x = 3$ (C) at $x = -\frac{1}{3}$

(D) at $x = \frac{1}{3}$ (E) it doesn't

23. Which of these polynomials is **not** a factor of $x^{10} - 1$?

(A) $x^5 - 1$

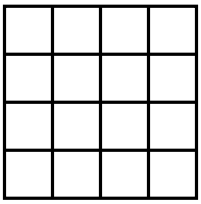
(B) $x^9 + x^8 + x^7 + \cdots + x + 1$

(C) $x^9 - x^8 + x^7 - \cdots + x - 1$

(D) $x^8 + x^6 + x^4 + x^2 + 1$

(E) $x^8 - x^6 + x^4 - x^2 + 1$

24. How many rectangles are in the accompanying figure?



(A) 30 (B) 50 (C) 60 (D) 81 (E) 100

25. Suppose that the temperature T at every point (x, y) in the plane is given by the formula

$$T = 1 - x^2 + 2y^2$$

Find the correct statement about the maximum and minimum temperature **along the line** $x + y = 1$.

(A) Min is -1. There is no max.

(B) Max is -1. There is no min.

(C) Max is 0. Min is -1.

(D) Max is 2. Min is 0.

(E) There is neither a max nor a min along the line.

2009 Answers / Level 3 Test

1. C
2. B
3. D
4. D
5. C
6. D
7. E
8. B
9. B
10. D
11. A
12. E
13. D
14. C
15. D
16. D
17. A
18. B
19. C
20. B
21. E
22. D
23. E
24. E
25. A