College of Charleston
Math Meet 2011
Written Test – Level 1

1. What is the greatest number of cubes, each 4 inches on a side, that would fit in a 2 foot by 3 foot by 5 foot rectangular box?

   (A) 90 (B) 120 (C) 480 (D) 570 (E) 810

2. A gumball machine contains 4 green and 6 pink gumballs, each with an equal probability of coming out of the machine when you put in a coin. You put in a coin and get one gumball, which you put in your pocket. Then, you put in another coin and put the second gumball in your pocket.

   What is the probability that both gum balls in your pocket are pink?

   (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{9}{25}$ (D) $\frac{1}{2}$ (E) $\frac{3}{5}$

3. For *how many* integers n is $n^2 + 6n - 27$ a positive prime number?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

4. What is the units digit of $3^{2011} \times 7^{2010}$?

   (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

5. Which of these choices lists the numbers in *increasing* order?

   (A) $2\sqrt{20}, 3\sqrt{5}, 9, 3\pi$ (B) $9, 3\pi, 3\sqrt{5}, 2\sqrt{20}$

   (C) $3\sqrt{5}, 9, 3\pi, 2\sqrt{20}$ (D) $3\sqrt{5}, 2\sqrt{20}, 9, 3\pi$

   (E) $2\sqrt{20}, 9, 3\sqrt{5}, 3\pi$

6. Let a, b, c and d be the positive whole numbers which satisfy

$$\frac{43}{9} = a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$$

   then what is $a \times b \times c \times d$?

   (A) 10 (B) 12 (C) 16 (D) 21 (E) 24

7. If $x^2 + \frac{1}{x^2} = \sqrt{104}$ then what is $\left| x^2 - \frac{1}{x^2} \right|$?

   (A) 10 (B) $\sqrt{12}$ (C) 14 (D) $\sqrt{15}$ (E) 17

8. The number 2011 can be written in binary form as 11111011011 using exactly 2 zeroes and 9 ones. Find the sum of the three smallest integers that are larger than 2011 and can also be written in binary form with 2 zeroes and 9 ones.

   (A) 6042 (B) 6050 (C) 6057 (D) 6065 (E) 6082

9. A line with slope $m = 2$ and a line with slope $m = 1/2$ meet at the point $(4, 6)$. What is the distance between their x-intercepts?

   (A) 5.5 (B) 6 (C) 7 (D) 8.25 (E) 9

10. A club consists of eight members, including Alonso, Betty, and Carl. How many ways are there to choose a president, secretary, and treasurer if Alonso cannot be president, and either Betty or Carl must be treasurer? (No one may hold more than one office.)

(A) 14 (B) 36 (C) 72 (D) 84 (E) 112

11. What is the sum of the solutions to the equation

$$|x-5|^2 + 2|x-5| = 3?$$

(A) 3 (B) 5 (C) 6 (D) 10 (E) 13

12. Which of these points lies *outside* the circle of radius 3 centered at $(2,1)$?

(A) $\left(\frac{4+3\sqrt{2}}{2}, \frac{2-3\sqrt{2}}{2}\right)$ (B) $\left(-\frac{1}{4}, \frac{5}{2}\right)$

(C) $\left(\frac{9}{2}, -\frac{3}{2}\right)$ (D) $\left(\frac{7}{2}, \frac{2+3\sqrt{3}}{2}\right)$

(E) $(0,3)$

13. Determine the value of $|(3-7)-(3-4)|$.

(A) 5 (B) -3 (C) -5

(D) 3 (E) none of these

14. A brother and sister were born on the same date but in different years. Six years ago the brother was $1/2$ his sister's age. Eleven years from now the brother will be $2/3$ his sister's age. How old is the sister now?

(A) 17 (B) 33 (C) 40 (D) 62 (E) 23

15. Find the coefficient of x^{50} in the polynomial

$$(1+x^2+x^4+x^6+\cdots+x^{100})(1+x^4+x^8+x^{12}+\cdots+x^{100}).$$

(A) 13 (B) 14 (C) 15 (D) 17 (E) 18

16. Express the number

$$0.\overline{21} = 0.21212121\ldots$$

as a rational number a/b in lowest terms. What is $a+b$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 189

17. A spinner has an equal probability of landing on each of seven spaces, labeled with the numbers one through seven. You are going to be playing a game in which you will spin it 70 times, each time betting in advance on whether it will land on an even or an odd number. Which strategy should you adopt in order to win the most times?

(A) There is no optimal strategy because it is random.

(B) Bet *odd* exactly 40 times and *even* exactly 30 times.

(C) Bet *odd* 35 times and *even* 35 times.

(D) Bet *even* only if the previous spin landed on an *odd* number.

(E) Bet on *odd* every time.

18. A radiator holds 5 gallons of coolant that is 30% antifreeze and 70% water. How much of the coolant should be drained out and replaced with pure antifreeze to bring it up to the recommended concentration of 50% antifreeze and 50% water?

(A) $2/3$ gallons (B) $4/5$ gallons (C) $8/3$ gallons

(D) $10/7$ gallons (E) $10/3$ gallons

19. Which statement is correct?

(A) If $x^2 > 0$ then $x > 0$. (B) If $x^2 > x$ then $x > 0$.

(C) If $x < 0$ then $x^2 > x$. (D) If $x^2 > x$ then $x < 0$.

(E) If $x < 1$ then $x^2 < x$.

20. When all of the students at her school were ranked according to their academic performance (with no ties), Sue happened to be both the 50th best and the 50th worst. How many students attend her school?

(A) 50 (B) 75 (C) 99 (D) 100 (E) 101

21. The intersection of a plane and a sphere can be which of the following?

I. a point II. a line III. a circle IV. the empty set

(A) I or II only (B) I, II or III only (C) II, III or IV only

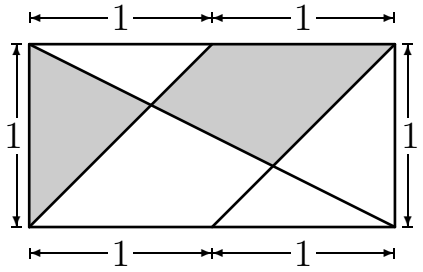
(D) I, III or IV only (E) III or IV only

22. A pyramid has a rectangular base with area 100 square inches, and its volume is 75 cubic inches. What is the height of the pyramid?

(A) 2.25 inches (B) 3.75 inches (C) 5.5 inches

(D) 12.5 inches (E) 25.25 inches

23. What is the area of the shaded region?



(A) $\frac{5}{6}$ (B) $\frac{4}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) 1

24. Jen and Ben bought identical boxes containing note paper and envelopes. Ben used three sheets to write each of his letters, putting each into an envelope, until he had no sheets left. In the end, he had 50 envelopes left over. Jen used just one sheet for each letter, putting each into an envelope, until she ran out of envelopes. In the end, she had 50 sheets of paper left over. How many sheets were in each box to start with?

(A) 150 (B) 125 (C) 120 (D) 100 (E) 80

25. Which of these formulas is equal to $\dfrac{\dfrac{a}{b}+\dfrac{b/a}{a-b}}{a}$ whenever $a \neq 0, b \neq 0$ and $a \neq b$?

(A) $\dfrac{a-b+b^2}{a^2b(a-b)}$ (B) $\dfrac{a^3-a^2b+b^2}{ab(a-b)}$ (C) $\dfrac{a^2-ab+b^2}{b(a-b)}$

(D) $\dfrac{a-b}{ab}$ (E) none of the above

2011 Answers / Level 1 Test

1. E
2. B
3. C
4. B
5. D
6. E
7. A
8. B
9. E
10. C
11. D
12. C
13. D
14. C
15. A
16. B
17. E
18. D
19. C
20. C
21. D
22. A
23. A
24. A
25. E