College of Charleston
Math Meet 2012
Written Test – Level 1

1. A sculpture at the College of Charleston's Halsey Art Gallery is made from one hundred stone blocks. Thirty-five of them are little blocks weighing 10 pounds each. Twenty-five of them are slightly bigger, 15 pound blocks. There are also twenty-five blocks that weigh 80 pounds each. Finally, there are fifteen 100 pound blocks.

   There are three common measures we can use to determine the "center" of this data: the mean, the median and the mode. Let m_1 be the mean of the weights of the blocks in the sculpture, let m_2 be the median weight, and let m_3 be the modal weight.

   Which of these sets of inequalities is true?

   (A) $m_1 < m_2 < m_3$ (B) $m_2 < m_3 < m_1$ (C) $m_3 < m_1 < m_2$

   (D) $m_3 < m_2 < m_1$ (E) $m_2 < m_1 < m_3$

2. A dish contains twenty wrapped chocolate candies. Eight of them are *dark* chocolate and twelve are *milk* chocolate. Ten of them have *yellow* wrappers and ten have *blue* wrappers. Only two of the candies with yellow wrappers are dark.

   I have randomly selected a candy from the dish and notice that it has a blue wrapper. What is the probability that it is *dark* chocolate?

   (A) .2   (B) .5   (C) .6   (D) .75   (E) .8

3. Simplify $\sqrt{\dfrac{8^8 + 4^8}{8^2 + 4^7}}$.

   (A) 4   (B) 8   (C) 16   (D) 32   (E) 128

4. How many ordered triples (x, y, z) are solutions to the equation $x + y + z = 9$ if x, y and z are positive integers?

   (A) 28   (B) 30   (C) 32   (D) 34   (E) 36

5. In front of me are four stacks labeled I, II, III and IV. Each stack contains 10 metal bars that look identical. However, in some of the stacks the bars all weigh 3 ounces each and in the other stacks the bars weigh only 2 ounces each. I weigh one bar from stack I, two bars from stack II, three bars from stack III, and four bars from stack IV together and find that the total weight is exactly 27 ounces.

   Which of the stacks have bars weighing 2 ounces?

   (A) I and II   (B) I and IV

   (C) III only   (D) II and III

   (E) not enough information to answer

6. Two cards are selected at random from a stack of four cards. We know that the cards in the stack are labeled "1", "2", "3" and "6". What is the probability that the *product* of the selected numbers is 6?

   (A) 0   (B) $1/3$ (C) $2/3$ (D) $1/4$ (E) $1/2$

7. Suppose $f(x) = mx + b$ is a linear function with positive slope and that $f(f(x)) = 4x - 15$. What is $m + b$?

   (A) -3 (B) -2 (C) -1 (D) 0   (E) 1

8. In front of me are four stacks labeled I, II, III and IV. Each stack contains 10 metal bars that look identical. However, in some of the stacks the bars all weigh 3 ounces each and in the other stacks the bars weigh only 2 ounces each. I weigh one bar from stack I, two bars from stack II, four bars from stack III, and eight bars from stack IV together and find that the total weight is exactly 36 ounces.
    Which of the stacks have bars weighing 2 ounces?

    (A) I, II and III (B) I and IV

    (C) II and III (D) II, III and IV

    (E) not enough information to answer

9. There are six positive integers that can evenly divide the number 2012. They are 1, 2, 4, 503, 1006 and 2012 itself. I have written these six numbers on the faces of a regular six-sided die. If I roll the die two times, what is the probability that the sum of the rolls is a divisor of 2012?

    (A) $1/36$ (B) $1/9$ (C) $1/6$ (D) $1/3$ (E) $1/2$

10. It is a proven fact that:

    > If 10^n is evenly divisible by the whole number d (for some number n) then the number a is evenly divisible by d if and only if the last n digits of a are evenly divisible by d.

    Which of the statements can we conclude is true by application of this principle?

    (A) 20070224064 is a multiple of 8

    (B) 125 is a factor of 2007022400575

    (C) 2007022400625 is not evenly divisible by 625

    (D) $899350032/32$ is a whole number

    (E) 48740384035 is divisible by 7^5

11. An aquarium has a rectangular base that measures 20 inches by 10 inches. What's the diameter of the smallest circular table that will hold the aquarium, if no part of the base can extend over the edge of the table?

    (A) 10 inches (B) 20 inches (C) 30 inches (D) $7\sqrt{7}$ inches (E) $10\sqrt{5}$ inches

12. Which of these is a mathematically accurate description of the significance of the formula $\dfrac{a + b - |a - b|}{2}$?

    (A) It is always equal to either a or b (whichever is *larger*).

    (B) It is always equal to either a or b (whichever is *smaller*).

    (C) It either has the value $a - b$ or $b - a$ (whichever is *larger*).

    (D) It either has the value $a - b$ or $b - a$ (whichever is *smaller*).

    (E) None of the above.

13. How many seconds are there in exactly six weeks?

    (A) $8!$ (B) $9!$ (C) $10!$ (D) $11!$ (E) $12!$

14. If A, B, and C are subsets of the set U, then
   - $A \cap B$ is the set of all elements that belong to both A **and** B,
   - $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
   - A^c is the set of all elements of U that are **not** in A.

   In the diagram, A, B and C are represented by circles and U is represented by a square.

   Which of the following expressions describes the region shaded in the diagram?
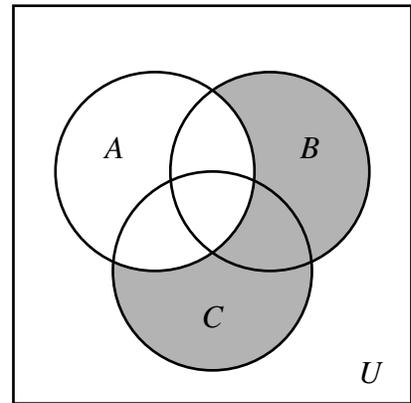
   (A) $(A \cap B)^c \cup C$　　　　(B) $A^c \cap (B \cup C)$　　　　(C) $(A \cup B^c) \cap C$

   (D) $A^c \cup (B \cap C)$　　　　(E) $(A \cup B) \cup C^c$

15. A rectangular box with no top has a width that is one inch less than the length. Its height is three inches less than the width. If the volume of the box is 20 cubic inches, what is its surface area?

   (A) 31 square inches　　　　(B) 5 square inches　　　　(C) 46 square inches

   (D) 38 square inches　　　　(E) $\sqrt{-46\pi}$ square inches

16. Peter has two thirds as many marbles as Henry. If Peter gets eight more marbles, he will have five sixths as many marbles as Henry. How many marbles do Peter and Henry have altogether?
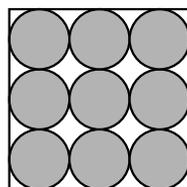
   (A) 60　　　　(B) 65　　　　(C) 70　　　　(D) 75　　　　(E) 80

17. Find the sum of the roots for $x^3 - 2x^2 - 3x + 6 = 0$.
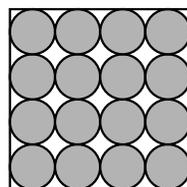
   (A) 0　　　　(B) 1　　　　(C) 2　　　　(D) 3　　　　(E) 4

18. A square, each side 4 units long, is packed with n adjacent rows of n adjacent congruent circles. (See figure.) What is the total area, in square units, of all of the circles inside the square?
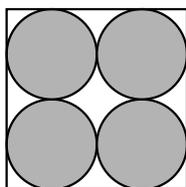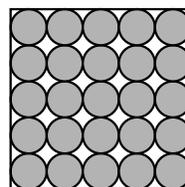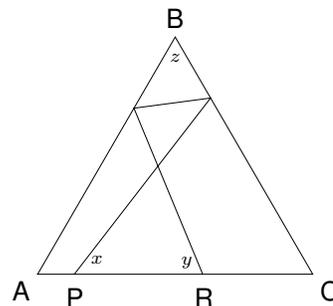

   $n = 2$　　　　$n = 3$　　　　$n = 4$　　　　$n = 5$

   (A) $2n\pi$　　　　(B) $\dfrac{8\pi}{1 + n^2}$　　　　(C) 2π

   (D) 4π　　　　(E) $\dfrac{16(n^2 - 2n + \pi)}{n^2}$

19. Inside triangle ABC, a ball is rolled from point P, bounces off two sides, and ends up at point R. What is the relationship between angles x, y, and z (in degrees)? (Note: ABC is not necessarily equilateral.)

(A) $x + y + z = 180$ (B) $x + y = 2z$ (C) $x = y = z$

(D) $x - z = y - z$ (E) none of the above

20. What is the smallest positive integer m for which there is an integer n such that $2010m + 2012n = 2$?

(A) 503   (B) 1005   (C) 2010   (D) 2011   (E) none of these

21. A boat is anchored in the middle of a still lake when its fuel tank begins to leak, causing a circular slick on the surface of the lake. If the radius of the slick is growing at a constant rate of 2 ft/min, approximately how much area will the slick cover after 1 hour?

(A) $(120)^2\pi$ square feet   (B) $(60)^2\pi$ square feet   (C) $(120)\pi$ square feet

(D) $(240)^2\pi$ square feet   (E) 240π square feet

22. The figure to the right can be completed to be a *Magic Square*. This means that you can fill in the empty boxes so that the sums of the entries of every row, every column and both diagonals are all equal. If the box is completed in this way, what entry goes in the *middle* box?

17		
2		
23	8	

(A) 2   (B) 3   (C) 7   (D) 11   (E) 14

23. A picture was duplicated using a photocopy machine. The copier had been set for a reduction to 80% of the original size. What percent will need to be used on the copier to obtain a full size copy from the reduced sized picture?

(A) 20% (B) 80% (C) 120% (D) 125% (E) none of these

24. How many real solutions are there to the equation $\sqrt{4x^2 - 1} - \sqrt{2x^2 + 3} = 1$?

(A) none   (B) 1   (C) 2   (D) 3   (E) more than 3

25. Circle C_1 has radius 2 and is centered at the point $(0, 0)$. Circle C_2 has radius 2 and is centered at the point $(0, 2)$. Find the distance between the points of intersection of C_1 and C_2.

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $2\sqrt{2}$ (D) $3\sqrt{2}$ (E) None of these

2012 Answers / Level 1 Test

1. D

2. C

3. D

4. A

5. E

6. B

7.	A	14.	B	21.	A
8.	B	15.	D	22.	E
9.	B	16.	E	23.	D
10.	A	17.	C	24.	C
11.	E	18.	D	25.	B
12.	B	19.	B		
13.	C	20.	B		