COLLEGE of CHARLESTON

MATH MEET 2016

Math Marathon

Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entries must be received or postmarked by February 19, 2016.

Marathon Problems

1. Find the longest geometric progression of positive integers whose sum is 2016.

2. There is a collection of lines in the plane, and there are 2016 points where at least two of these lines intersect. What is the minimum number of lines in such a collection?

3. The random number generator on a computer is used to generate three numbers a, b, c uniformly in the interval $[0,1]$. What is the probability that the line with equation $a\,x + b\,y = c$ intersects the unit circle $x^2 + y^2 = 1$?

4. A circle centered at the origin is tangent to the parabola $y = k - x^2$ at two distinct points which are k units apart. Find all possible values for the positive constant k.

5. Let f denote the rational function

$$f(x) = \frac{a\,x + b}{c\,x + d},$$

where a, b, c, d are positive constants, and let g denote the 2016-fold compositional iterate of f, that is,

$$g = \underbrace{f \circ f \circ \cdots \circ f}_{\text{2016 copies}}.$$

If g has a vertical asymptote at $x = A$ and satisfies $g(0) = B$, express the product AB as a function of a, b, c, d.