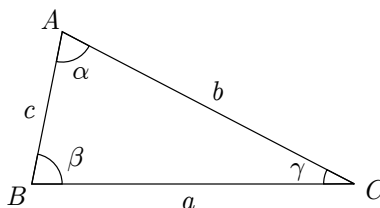




## Please Learn Unusual Trigonometry Obsessively

The residents of Pluto have studied the properties of shapes on their homeworld, resulting in the so-called plutometry, the Plutonian version of geometry. Let  $\triangle ABC$  be a triangle with vertices  $A$ ,  $B$ , and  $C$ . Let  $\alpha$  be the angle  $\angle CAB$ ,  $\beta$  be the angle  $\angle ABC$ , and  $\gamma$  be the angle  $\angle BCA$  (all of which are measured in radians). Let  $a$  be the length of side  $BC$ ,  $b$  be the length of the side  $AC$ , and  $c$  be the length of the side  $AB$  (all of which are measured in kilometers).



Pluto's radius is 1187 km, so the number 1187 appears frequently in plutometry. Here are some of the plutometric results about triangles:

- The angles in a triangle always add up to **more** than  $\pi$ .
- The area of triangle  $\triangle ABC$  is  $(1187)^2(\alpha + \beta + \gamma - \pi)$ .
- Law of Cosines:  $\cos \frac{c}{1187} = \cos \frac{a}{1187} \cos \frac{b}{1187} + \sin \frac{a}{1187} \sin \frac{b}{1187} \cos \gamma$
- Law of Sines:  $\frac{\sin \frac{a}{1187}}{\sin \alpha} = \frac{\sin \frac{b}{1187}}{\sin \beta} = \frac{\sin \frac{c}{1187}}{\sin \gamma}$
- The Pythagorean Theorem (a special case of the Law of Cosines):  
if  $\gamma$  is a right angle (so  $\gamma = \frac{\pi}{2}$ ), then  $\cos \frac{c}{1187} = \cos \frac{a}{1187} \cos \frac{b}{1187}$ .

There are three plutonian settlements: Alan, Burney, and Clyde. The distance from Alan to Burney is 700 km. The distance from Burney to Clyde is 800 km. The distance from Alan to Clyde is 900 km.

In the following questions, all angles are measure in radians. Give your answers as decimal approximations accurate to within 0.1%.

- Find all three angles of the triangle whose vertices are placed at Alan, Burney, and Clyde.

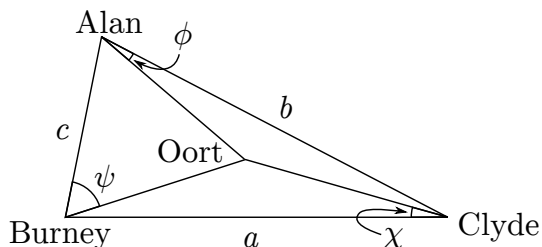
$\angle$ Alan:

$\angle$ Burney:

$\angle$ Clyde:

- What is the area of the triangle? Area:   $\text{km}^2$

- The plutocrats who run the government want to build a capital city Oort which is equidistant from the other three settlements. How large are the angles  $\phi$ ,  $\psi$ , and  $\chi$  indicated in the picture (which is not drawn to scale, and is drawn using primitive Earth geometry).



$\phi$ :

$\psi$ :

$\chi$ :

- How far will Oort be from Alan, Burney, and Clyde?

Distance:  km

- A fourth settlement, Dwarf, is to be founded. It must be exactly as far from Oort as the other three settlements are. Also, to give Dwarf as much space as possible, it must be placed so it is as far as possible from the nearest of Alan, Burney, and Clyde. How far should Dwarf be from the nearest of Alan, Burney, and Clyde?

Distance:  km

