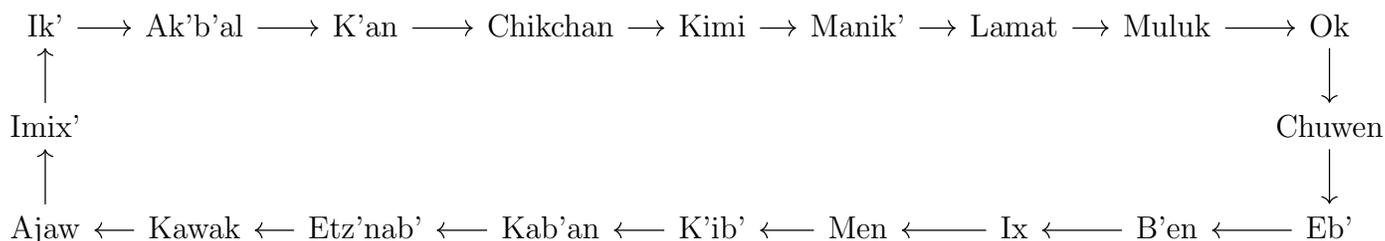




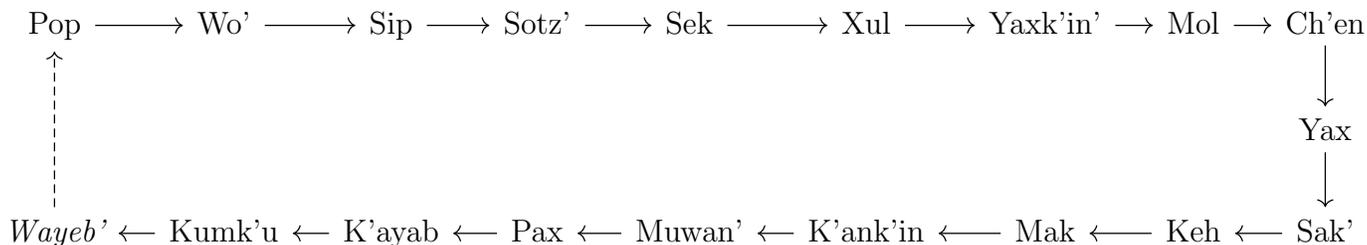
Settlers of Yucatan

In some ways, the Maya calendar is more complicated than our modern calendar (called the Gregorian calendar), and in some ways it's simpler. Each day is identified by four pieces of information. Two of these pieces of information are collectively called the Tzolk'in.

- The first part of the Tzolk'in is a number. These continually repeat a cycle from 1 to 13, changing every day.
- The second part of the Tzolk'in is a name (like our 7 days of the week). These cycle through a sequence of 20 names, advancing by one name each day, as shown below.

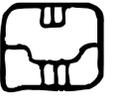


The other two pieces of information are called the Haab'. There are 18 months (starting with Pop), each with 20 days. Since that's only 360 days, while a year should be 365 days, there's a 19th month (called Wayeb') at the end, with only 5 days. Within each month, the day numbers start at 0 (not 1) and end at either 19 or 4, depending on whether the month is Wayeb'. The month names are



One would typically write the Tzolk'in information (number followed by name) and then the Haab' information (number followed by name). Some examples:

- The Maya believed the world was created on 4 Ajaw 8 Kumk'u.
- We're not exactly sure which days on the Maya calendar correspond to which days on the Gregorian calendar. One common translation system says that today is 9 B'en 16 Pax. Tomorrow will be 10 Ix 17 Pax.
- The day after 13 Chikchan 8 Sek is 1 Kimi 9 Sek.
- The day after 7 K'ib' 4 Wayeb' is 8 Kab'an 0 Pop (which starts a new Haab' cycle)



1. How many different Tzolk'in are possible?



2. Maya dates eventually repeat. Today is 9 B'en 16 Pax (February 11, 2017). How many days will go by before the next 9 B'en 16 Pax?



3. Suppose that we know that the Tzolk'in is 5 Lamat and the Haab' month is Sip. What are the possible values for the Haab' day number?



4. If Wayeb' starts on 3 Manik', what will the Tzolk'in be at the start of the next Wayeb' after that?



5. Starting from 12 Ajaw 18 K'ayab, how many days go by before the first 3 Lamat 1 Wo'?



6. Let's call a day *symmetric* if its Tzolk'in number is equal to its Haab' number. For example, 6 Etz'nab' 6 Mol is symmetric, while 2 Eb' 5 Sotz' isn't. The number of symmetric days in a full Haab' cycle (from 0 Pop through to 4 Wayeb'), varies from one full Haab' cycle to another, depending on the Tzolk'in of 0 Pop.

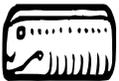
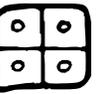


What's the smallest possible number of symmetric days in a full Haab' cycle?

What's the largest it could possibly be?



7. The Maya year (the Haab' cycle) has exactly 365 days. The amount of time that it takes for the Earth to go around the sun is actually a little more than 365 days. The ancient Roman calendar (called the Julian calendar, after Julius Casesar) fixed this problem by adding an extra day (February 29th, called a leap day) every four years. Some years would be 365 days long, and others would be 366 days long, but the length of the year averaged out to 365.25 days, which was closer to correct. However, 365.25 is a tiny bit too large, so in 1582, Europeans started switching to a new calendar (called the Gregorian calendar, after Pope Gregory XIII). Three times out of every 400 years, the Gregorian calendar skips a leap day (the last time was in the year 1900). This makes the average length of the year 365.2425 days.



Let's try to fix the Maya calendar. Suppose that whenever Wayeb' starts on a Tzolk'in number of 1, 5, or 9, we add an extra day to Wayeb'. For example, if the first day of Wayeb' is 5 Kab'an 0 Wayeb', then the last day of Wayeb' would be 10 Ik' 5 Wayeb'.



With this fix, how many days are in the average Haab' cycle?

