COLLEGE of CHARLESTON

MATH MEET 2018

Math Marathon

Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entries must be received or postmarked by February 12, 2018.

Marathon Problems

1. Find all positive integers n less than 2018 which have the property that n^3 is congruent to n modulo 2018.

2. Three positive integers a, b, and c are chosen so that the graph of the function

$$f(x)=\sqrt{ax-b}-c$$

has exactly one point of intersection with its inverse function. What is the smallest possible value of $a+b+c$?

3. The random number generator on a computer is used to generate three real numbers a, b, and c uniformly and independently in the interval $[0,x]$, where $x>0$. What is the probability that the circle with center (a,b) and radius c intersects the circle with center $(0,0)$ and radius c?

4. Express 2018 as a sum of distinct positive integer squares, using as many distinct squares as possible.

5. Three people each toss a coin repeatedly until it lands heads, and count the number of tosses it takes them. What is the probability that each of the three people require the same number of tosses?