MATH MEET 2019

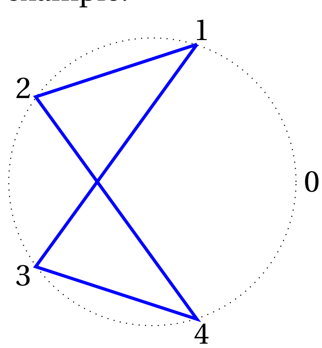
Basket Plot All-Day Sprint

This year's tee shirt features diagrams that I've taken to calling basket plots. There are two kinds.

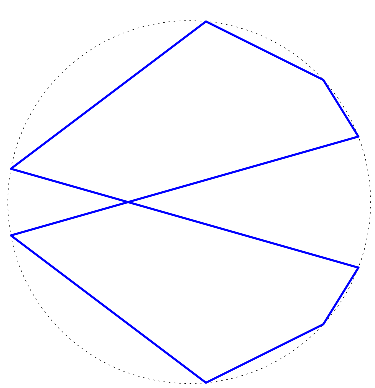
First, let $n > 1$ be an integer, and let $1 < a < n$ be an integer. If you look at the sequence of integers $1, a, a^2, a^3, \ldots$, all modulo n, and reduce everything to its standard representation, sooner or later the sequence has to repeat itself. For example, working modulo $n = 5$ with $a = 2$, the sequence of powers of 2 reduces to

$$1, 2, 2^2 \equiv 4, 2^3 \equiv 8 \equiv 3, 2^4 \equiv 6 \equiv 1, \ldots$$

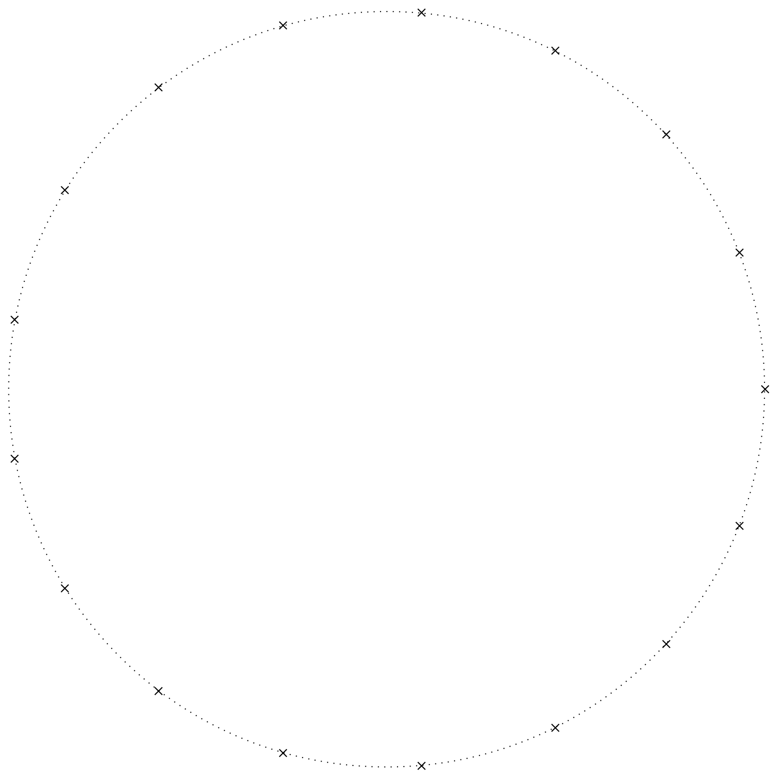
To draw a picture of this sequence, called the orbit of 2, draw a circle and distribute points labeled $0, 1, \ldots 4$ evenly around it. Then connect them in order $1, 2, 4, 3, 1, \ldots$. Continuing our example:



Here is the orbit of 2 mod 17, without the numbers:



1. Draw the orbit of 5 modulo 17:



FYI: The diagrams on this year's tee shirt are made by working with complex integers modulo 7. The diagram at location (n, m) on the tee shirt is the orbit of $n + mi$ mod 7, where $i = \sqrt{-1}$ is the imaginary unit. Each complex integer is equivalent mod 7 to a number $x + yi$ where $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$. These orbits are draw on a lattice rather than a circle.