The 2019 Sprint

Instructions: Write the exact answer to each question in the corresponding blank. Remember that the winners in this event are those participants who answer the most questions correctly *in a row* beginning with the first question. So, try to get as far as you can without making a mistake!

1. Find the prime factorization of 2019.
   
   1. __________________________

2. Consider the repeating decimal
   
   \[ \overline{0.2019} = 0.20192019201920192019 \ldots \]

   Write this number as a fraction in lowest terms.

   2. __________________________

3. When fully expanded, what is the units digit of \( 2^{(23^{19})} \)?

   3. __________________________

4. Find the smallest positive integer solution to the equation

   \[
   \frac{1 + 3 + 5 + 7 + \cdots + (2n - 1)}{2 + 4 + 6 + 8 + \cdots + 2n} = \frac{2018}{2019}
   \]

   4. __________________________

5. We use \( n! \) to mean the product \( 1 \cdot 2 \cdot 3 \cdots n \). For example, \( 4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24 \). Determine the number of zeros at the end of \( 2019! \) when expanded.

   5. __________________________

6. Evaluate the following sum and simplify your answer:

   \[
   \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \cdots + \frac{1}{2018 \cdot 2019}
   \]

   6. __________________________

7. Given the sequence \( \{x_n\} \) defined by \( x_{n+1} = \frac{1 + x_n \sqrt{3}}{\sqrt{3} - x_n} \) with \( x_1 = 1 \), compute the value of

   \[ x_2 + x_{23} + x_{2019} \]

   7. __________________________
The grading for the Timed Sprints is unusual! Your grade will be the number of questions answered correctly, starting with the first question, before you make a mistake. For example, if you only answer questions 1-4 correctly and questions 7-13 correctly, your grade will be a “4” since you did not get question 5 right. You will have a limited amount of time to work on the sprint. Your paper will be collected at the end of this period.

By my signature below I certify that all of the work completed on this sprint is my own.