

The 2019 Sprint

Instructions: Write the exact answer to each question in the corresponding blank. Remember that the winners in this event are those participants who answer the most questions correctly *in a row* beginning with the first question. So, try to get as far as you can without making a mistake!

1. Find the prime factorization of 2019.

1. _____

2. Consider the repeating decimal

$$0.\overline{2019} = 0.20192019201920192019\dots$$

Write this number as a fraction in lowest terms.

2. _____

3. When fully expanded, what is the units digit of $2^{(2^{3^{19}})}$?

3. _____

4. Find the smallest positive integer solution to the equation

$$\frac{1 + 3 + 5 + 7 + \dots + (2n - 1)}{2 + 4 + 6 + 8 + \dots + 2n} = \frac{2018}{2019}$$

4. _____

5. We use $n!$ to mean the product $1 \cdot 2 \cdot 3 \cdots n$. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$. Determine the number of zeros at the end of $2019!$ when expanded.

5. _____

6. Evaluate the following sum and simplify your answer:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \dots + \frac{1}{2018 \cdot 2019}$$

6. _____

7. Given the sequence $\{x_n\}$ defined by $x_{n+1} = \frac{1 + x_n \sqrt{3}}{\sqrt{3} - x_n}$ with $x_1 = 1$, compute the value of

$$x_2 + x_{23} + x_{2019}$$

7. _____

College of Charleston Math Meet 2019

The 2019 Sprint

Name (please print): _____

School: _____

The grading for the Timed Sprints is unusual! Your grade will be the number of questions answered correctly, starting with the first question, before you make a mistake. For example, if you only answer questions 1-4 correctly and questions 7-13 correctly, your grade will be a "4" since you did not get question 5 right. You will have a limited amount of time to work on the sprint. Your paper will be collected at the end of this period.

By my signature below I certify that all of the work completed on this sprint is my own.