COLLEGE of CHARLESTON

MATH MEET 2019

Math Marathon

Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- The cover paper for each entry must have the following information: (This may be turned in the day of the Math Meet if submitted electronically and not mailed.) Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entries must be received or postmarked by February 12, 2019.

1. The integers 2019 and 2020 each have exactly four odd divisors. Among the first 2020 positive integers, how many have exactly three odd divisors?

2. Find a positive constant b so that the minimum distance from the graph of the function $f(x)=\sqrt{x-b}$ to the graph of its inverse function is b units.

3. Alice tosses a coin until it lands heads and counts the number of trials it takes. Bob tosses a die until it lands 6 and counts the number of trials it takes. What is the probability that the number of trials Alice requires is fewer than the number of trials Bob requires?

4. Evaluate the sum

$$\sum_{n=1}^{2019} \frac{3n+2}{n^3+3n^2+2n}.$$

5. The positive integer 2019 can be written as a sum or difference of distinct powers of 2; that is, it can be expressed in the form $2019=\sum_{i=0}^{N} a_i 2^i$ where each coefficient a_i lies in $\{-1,0,1\}$. Find such a representation which uses the fewest nonzero coefficients. Do the same for powers of 3, that is, express $2019=\sum_{i=0}^{N} a_i 3^i$ with each $a_i \in \{-1,0,1\}$. Finally, find all positive integer bases b for which such a representation $2019=\sum_{i=0}^{N} a_i b^i$ with each $a_i \in \{-1,0,1\}$ is possible.