COLLEGE of CHARLESTON

MATH MEET 2020

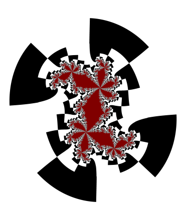
Math Marathon

Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- The cover paper for each entry must have the following information: Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification. This may be turned in the day of the Math Meet if submitted electronically and not mailed.
- All entries must be received or postmarked by February 14, 2020.



1. Find the minimum perimeter of a right triangle with integer sides and hypotenuse 2020.

2. A fair coin is tossed until either two consecutive heads or two consecutive tails are obtained. What is the expected number of coin tosses required?

3. For a positive integer m let M_m denote the positive integer whose decimal representation consists of a string of m 2s followed by 020. For example, $M_4 = 2222020$ is the date of this year's math meet. Determine all values of m for which M_m is divisible by 2020.

4. Find a nonzero quadratic polynomial $f(x) = ax^2 + bx + c$ with integer coefficients such that $f(\cos(\frac{\pi}{5})) = 0$.

5. A random number generator generates two numbers a, b uniformly and independently in the interval $[0, 10]$. What is the probability that the line with equation $x + ay = b$ intersects the ellipse with equation $x^2 + ay^2 = b$?