

# Math Marathon

## Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- Include a cover sheet for each entry with the following information: Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to [mathmeet@cofc.edu](mailto:mathmeet@cofc.edu) or mailed to the following address  
Math Meet (Marathon)  
Department of Mathematics  
College of Charleston  
66 George Street  
Charleston, SC 29424
- All entries must be received or postmarked by February 4, 2022.

## Problems

1. A family of distinct lines divides the plane into several regions so that the number of bounded regions exceeds the number of unbounded regions by 2022. What is the minimum number of lines?

2. A right triangle has integer sides and hypotenuse 2022. Find its perimeter.

3. The function

$$\underbrace{f \circ f \circ \cdots \circ f}_{2022 \text{ copies}}$$

has a vertical asymptote at  $x = A$  and a horizontal asymptote at  $y = B$ , where

$$f(x) = \frac{1}{x+1}.$$

Find  $A + B$ .

4. A six-sided die is cast repeatedly until the same number appears on consecutive throws. What is the expected value of the number of throws required?

5. Find the equation of the largest circle contained in the region

$$\{(x, y) : x^2 \leq y \leq \sqrt{x}\}.$$